Funções Trigonométricas 2
RESOLUÇÃO

1.1

$$\frac{\cos x(\cos x - \operatorname{sen} x)}{1 - \operatorname{sen} 2x} = \frac{\cos x(\cos x - \operatorname{sen} x)}{1 - 2\operatorname{sen} x \cos x}$$

$$= \frac{\cos x(\cos x - \operatorname{sen} x)}{\cos^2 x + \operatorname{sen}^2 x - 2\operatorname{sen}\cos x} =$$

$\cos^2 x + \operatorname{sen}^2 x = 1$

$$= \frac{\cos x(\cos x - \operatorname{sen} x)}{\cos^2 x - 2\operatorname{sen} x + \operatorname{sen}^2 x}$$

$a^2 - 2ab + b^2 =$
$= (a-b)^2$

$$= \frac{\cos x\,\cancel{(\cos x - \operatorname{sen} x)}}{(\cos x - \operatorname{sen} x)^{\cancel{2}}} = \frac{\cos x}{\cos x - \operatorname{sen} x} \quad \text{c.q.m}$$

1.2

$$\frac{\cos(2x) + 1 + \cos^2 x}{\cos x} = \frac{\cos^2 x - \operatorname{sen}^2 x + 1 + \cos^2 x}{\cos x} =$$

$$= \frac{2\cos^2 x + 1 - \operatorname{sen}^2 x}{\cos x} = \frac{2\cos^2 x + \cos^2 x}{\cos x} =$$

$$= \frac{3\cos^{\cancel{2}} x}{\cancel{\cos x}} = 3\cos x \quad \text{c.q.m.}$$

1.3 $\dfrac{2\tan x}{1+\tan^2 x} = \dfrac{2\tan x}{\dfrac{1}{\cos^2 x}} =$ $\quad 1+\tan^2 x = \dfrac{1}{\cos^2 x}$

$= 2\tan x \cdot \cos^2 x = 2\,\dfrac{\operatorname{sen} x}{\cancel{\cos x}} \cdot \cos^{\cancel{2}} x =$

$= 2\operatorname{sen} x \cos x = \operatorname{sen} 2x$ c.q.m.

1.4 $\cos(6x) + \operatorname{sen}^4(3x) = \cos(2\cdot 3x) + \operatorname{sen}^4(3x)$

$= \cos^2(3x) - \operatorname{sen}^2(3x) + \operatorname{sen}^4(3x) =$

$= \cos^2(3x) + \operatorname{sen}^2(3x)\left(\operatorname{sen}^2(3x) - 1\right) =$

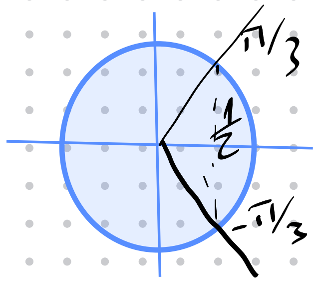
$= \cos^2(3x) - \operatorname{sen}^2(3x)\left(1 - \operatorname{sen}^2(3x)\right) =$

$= \cos^2(3x) - \operatorname{sen}^2(3x)\cos^2(3x) =$

$= \cos^2(3x)\left(1 - \operatorname{sen}^2(3x)\right) = \cos^2(3x)\cos^2(3x)$

$= \cos^4(3x)$ c.q.m.

2.1 $\cos^2 x - \sin^2(x) = \frac{1}{2} \Leftrightarrow$



$\Leftrightarrow \cos 2x = \frac{1}{2} \Leftrightarrow$

$\Leftrightarrow 2x = \frac{\pi}{3} + 2k\pi \vee 2x = -\frac{\pi}{3} + 2k\pi, \ k \in \mathbb{Z}$

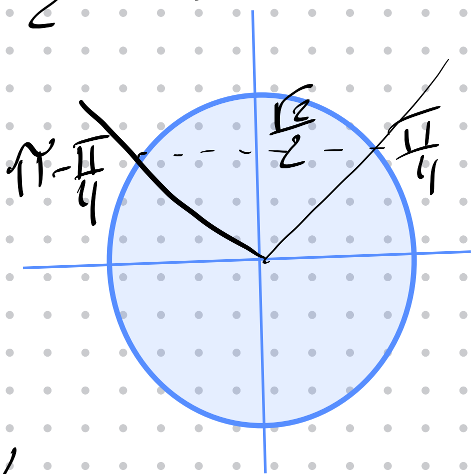
$\Leftrightarrow x = \frac{\pi}{6} + k\pi \vee x = -\frac{\pi}{6} + k\pi, \ k \in \mathbb{Z}$

2.2 $\cos x - \sqrt{3} \sin x = \sqrt{2} \Leftrightarrow$

$\Leftrightarrow \frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x = \frac{\sqrt{2}}{2} \Leftrightarrow$

$\Leftrightarrow \sin \frac{\pi}{6} \cos x - \cos \frac{\pi}{6} \sin x = \frac{\sqrt{2}}{2} \Leftrightarrow$

$\Leftrightarrow \sin\left(x - \frac{\pi}{6}\right) = \frac{\sqrt{2}}{2} \Leftrightarrow$



$\Leftrightarrow x - \frac{\pi}{6} = \frac{\pi}{4} + 2k\pi$

$\vee$

$x - \frac{\pi}{6} = \pi - \frac{\pi}{4} + 2k\pi, \ k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{\pi}{6} + \frac{\pi}{4} + 2k\pi \vee x = \frac{\pi}{6} + \frac{3\pi}{4} + 2k\pi, \ k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{5\pi}{12} + 2k\pi \vee x = \frac{11\pi}{12} + 2k\pi, \ k \in \mathbb{Z}$

2.3 $\cos(2x) - 11\cos x + 6 = 0 \Leftrightarrow$

$\Leftrightarrow \cos^2 x - \text{sen}^2 x - 11\cos x + 6 = 0 \Leftrightarrow$

$\Leftrightarrow \cos^2 x - 11\cos x + 5 + 1 - \text{sen}^2 x = 0 \Leftrightarrow$

$\Leftrightarrow \cos^2 x - 11\cos x + 5 + \cos^2 x = 0 \Leftrightarrow$

$\Leftrightarrow 2\cos^2 x - 11\cos x + 5 = 0$

Seja $y = \cos^2 x$

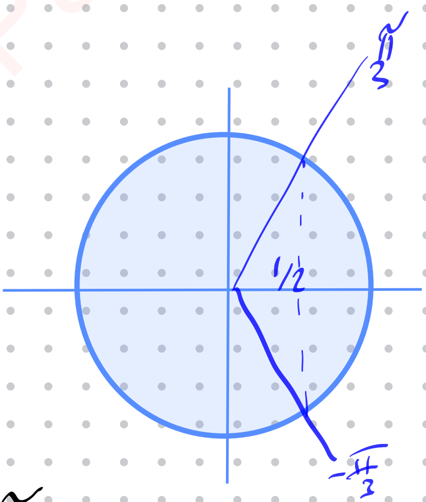
$\Leftrightarrow 2y^2 - 11y + 5 = 0 \Leftrightarrow$

$\Leftrightarrow y = \dfrac{11 \pm \sqrt{121 - 40}}{4}$

$\Leftrightarrow y = 5 \vee y = \dfrac{1}{2}$

$\Leftrightarrow \underbrace{\cos x = 5}_{\text{Impossível em } \mathbb{R}} \vee \cos x = \dfrac{1}{2}$

$-1 \leq \cos x \leq 1$



$\Leftrightarrow x = \dfrac{\pi}{3} + 2k\pi \vee x = -\dfrac{\pi}{3} + 2k\pi, k \in \mathbb{Z}$

3. $g(x) = \cos(2x) - 2\,\text{sen}\,x$

$\alpha \in \,]-\pi, 0[$

$\cos\alpha = \dfrac{\sqrt{3}}{2}$

$g(\alpha) = \cos(2\alpha) - 2\,\text{sen}\,\alpha =$

$= \cos^2\alpha - \text{sen}^2\alpha - 2\,\text{sen}\,\alpha =$

$= \cos^2\alpha - (1 - \cos^2\alpha) - 2\,\text{sen}\,\alpha =$

$= 2\cos^2\alpha - 1 - 2\cos\left(-\dfrac{\pi}{2} - \alpha\right)$

$= 2\left(\dfrac{\sqrt{3}}{2}\right)^2 - 1 - 2\cos\left(-\dfrac{2\pi}{3}\right) = \dfrac{6}{4} - 1 - 2\left(-\dfrac{1}{2}\right) = \dfrac{3}{2}$

C.A.

em $]-\pi, 0[$

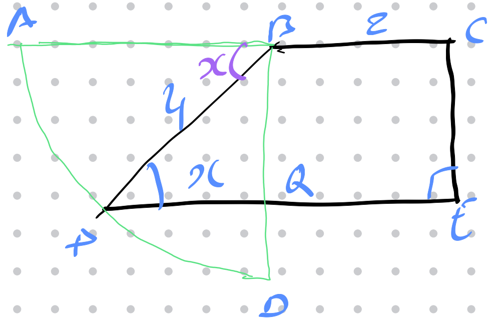
$\cos\alpha = \dfrac{\sqrt{3}}{2}$

$\cos\alpha = \cos\dfrac{\pi}{6}$

$\alpha = -\dfrac{\pi}{6}$

4.

$\overline{PB} = 4 \qquad \overline{BC} = 2$



$$A_{[PBCE]} = \frac{\overline{BC} + \overline{PE}}{2} \cdot \overline{CE}$$

$$= \frac{2 + (2 + 4\cos x)}{2} \cdot 4\,\text{sen}\,x$$

$$= \frac{16\,\text{sen}\,x + 16\cos x\,\text{sen}\,x}{2}$$

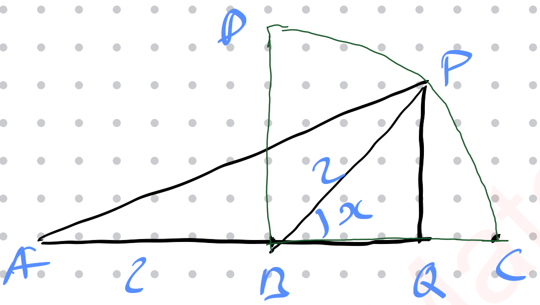
$$= 8\,\text{sen}\,x + 8\cos x\,\text{sen}\,x$$

$$= 8\,\text{sen}\,x + 4 \cdot 2\cos x\,\text{sen}\,x$$

$$= 8\,\text{sen}\,x + 4\,\text{sen}\,2x \quad , \; x \in \left]0, \frac{\pi}{2}\right[$$

c.q.m.

5.



$$A_{[APQ]} = \frac{\overline{AQ} \cdot \overline{PQ}}{2}$$

$\overline{AQ} = \overline{AB} + \overline{BQ} = 2 + 2\cos x$

$\overline{PQ} = 2\,\text{sen}\,x$

$$A(x) = \frac{(2 + 2\cos x)\,\cancel{2}\,\text{sen}\,x}{\cancel{2}} =$$

$$= 2\,\text{sen}\,x + 2\cos x\,\text{sen}\,x$$

$$= 2\,\text{sen}\,x + \text{sen}\,(2x) \qquad \text{c.q.m.}$$

6. $f(x) = x \operatorname{sen} x + \cos x$

6.1 $f(x) = x^2 \Leftrightarrow x \operatorname{sen} x + \cos x = x^2$

$\Leftrightarrow x \operatorname{sen} x + \cos x - x^2 = 0$

Seja $h(x) = x \operatorname{sen} x + \cos x - x^2$

h x é contínua em $\mathbb{R}$, pois é a adição de 3 funções contínuas em $\mathbb{R}$, em particular é contínua em $[0, \pi]$

$h(0) = -\pi \operatorname{sen}(0) + \cos(0) - (0)^2 =$

$= 1$

$h(\pi) = \pi \operatorname{sen} \pi + \cos \pi - (\pi)^2 =$

$= \pi \times 0 - 1 - \pi^2 = -1 - \pi^2$

Assim, $h(0) \times h(\pi) < 0$, e h é contínua em $[0, \pi]$, então pelo corolário do Teorema de Bolzano-Cauchy $\exists x \in]0, \pi[: f(x) = 0$

6.2 f é a soma de duas funções contínuas em $\mathbb{R}$, logo é contínua em $\mathbb{R}$, em particular é contínua em $[-\frac{\pi}{2}, \pi]$

$$f\left(-\frac{\pi}{2}\right) = -\frac{\pi}{2} \operatorname{sen}\left(-\frac{\pi}{2}\right) + \cos\left(-\frac{\pi}{2}\right)$$
$$= -\frac{\pi}{2}(-1) + 0 = \frac{\pi}{2} > 0$$

$$f(\pi) = \pi \operatorname{sen} \pi + \cos \pi =$$
$$= \pi(0) + (-1) = -1 < 0$$

Assim como f é contínua em $[-\frac{\pi}{2}, \pi]$ e $f(\pi) < 0 < f(-\frac{\pi}{2})$

Pelo Teorema de Bolzano-Cauchy

$$\exists x \in \left]-\frac{\pi}{2}, \pi\right[: f(x) = 0$$