1. Número de Neper:

1.1. $\lim\left(1+\frac{5}{n}\right)^n = e^5$

1.2. $\lim\left(1+\frac{5}{2n}\right)^n = \lim\left[\left(1+\frac{5}{2n}\right)^{2n}\right]^{\frac{1}{2}} = \left(e^5\right)^{\frac{1}{2}} = e^{\frac{5}{2}}$

1.3. $\lim\left(1-\frac{1}{n}\right)^n = \lim\left(1+\frac{-1}{n}\right)^n = e^{-1} = \frac{1}{e}$

1.4. $\lim\left(1+\frac{\pi}{n}\right)^{n+1} = \lim\left(1+\frac{\pi}{n}\right)^n\left(1+\frac{\pi}{n}\right) = e^{\pi}\times(1+0) = e^{\pi}$

1.5. $\lim\left(\frac{n+7}{n+4}\right)^n = \lim\left(\frac{\cancel{n}\left(1+\frac{7}{n}\right)}{\cancel{n}\left(1+\frac{4}{n}\right)}\right)^n = \frac{\lim\left(1+\frac{7}{n}\right)^n}{\lim\left(1+\frac{4}{n}\right)^n} = \frac{e^7}{e^4} = e^3$

1.6. $\lim\left(\frac{n+4}{n+2}\right)^{2n} = \lim\left(\frac{\cancel{n}\left(1+\frac{4}{n}\right)}{\cancel{n}\left(1+\frac{2}{n}\right)}\right)^{2n} = \left[\frac{\lim\left(1+\frac{4}{n}\right)^n}{\lim\left(1+\frac{2}{n}\right)^n}\right]^2 = \left(\frac{e^4}{e^2}\right)^2 = \left(e^2\right)^2 = e^4$

1.7. $\lim\left(\frac{n+1}{3n+2}\right)^n$

$$\lim\left(\frac{n+1}{3n+2}\right)^n = \lim\left(\frac{\cancel{n}\left(1+\frac{1}{n}\right)}{3\cancel{n}\left(1+\frac{2}{3n}\right)}\right)^n = \frac{\lim\left(1+\frac{1}{n}\right)^n}{\lim\left(1+\frac{2}{3n}\right)^n} = \frac{e}{\lim\left(\left(1+\frac{2}{3n}\right)^{3n}\right)^{\frac{1}{3}}} = \frac{e}{e^{\frac{2}{3}}} = e^{\frac{1}{3}}$$

1.8. $\lim\left(1-\frac{5}{n^2}\right)^n$

$$\lim\left(1-\frac{5}{n^2}\right)^n = \lim\left(1+\frac{-5}{n^2}\right)^n = \lim\left(\left(1+\frac{-5}{n^2}\right)^{n^2}\right)^{\frac{1}{n}} = \left(e^{-5}\right)^0 = 1$$

1.9. $\lim\left(1-\frac{1}{n+1}\right)^n$

$$\lim\left(1-\frac{1}{n+1}\right)^n=\lim\left(\frac{n+1-1}{n+1}\right)^n=\lim\left(\frac{n}{n+1}\right)^n=\lim\left(\frac{n+1}{n}\right)^{-n}=\lim\left(1+\frac{1}{n}\right)^{-n}=$$
$$=\lim\left(\left(1+\frac{1}{n}\right)^n\right)^{-1}=e^{-1}$$

1.10. $\lim\left(1-\frac{1}{(n+1)^2}\right)^n$

$$\lim\left(1-\frac{1}{(n+1)^2}\right)^n=\lim\left[\left(1+\frac{-1}{(n+1)^2}\right)^{(n+1)^2}\right]^{\frac{n}{(n+1)^2}}=\left(e^{-1}\right)^{\lim\frac{n}{n^2+2n+1}}=\left(e^{-1}\right)^0=1$$

2. Funções exponenciais e logarítmicas

2.1. $\lim\limits_{x\to0}\frac{x}{e^x-1}=\frac{1}{\lim\limits_{x\to0}\frac{e^x-1}{x}}=\frac{1}{1}=1$

2.2. $\lim\limits_{x\to0}\frac{2e^x-2}{7x}=\frac{2}{7}\lim\limits_{x\to0}\frac{e^x-1}{x}=\frac{2}{7}\times1=\frac{2}{7}$

2.3. $\lim\limits_{x\to0}\frac{e^{2x}-1}{x}=2\lim\limits_{x\to0}\frac{e^{2x}-1}{2x}\underset{\substack{x\to0\\2x\to0}}{=}2\lim\limits_{x\to0}\frac{2^{2x}-1}{2x}=2\times1=2$

2.4. $\lim\limits_{x\to0}\frac{e^x-1}{e^{4x}-1}=\lim\limits_{x\to0}\frac{\frac{e^x-1}{x}}{\frac{e^{4x}-1}{x}}=\frac{\lim\limits_{x\to0}\frac{e^x-1}{x}}{4\lim\limits_{x\to0}\frac{e^{4x}-1}{4x}}\underset{\substack{x\to0\\4x\to0}}{=}\frac{1}{4\lim\limits_{4x\to0}\frac{e^{4x}-1}{4x}}=\frac{1}{4\times1}=\frac{1}{4}$

2.5. $\lim\limits_{x\to0}\frac{e^x-e^{-x}}{x}=\lim\limits_{x\to0}\frac{e^x-\frac{1}{e^x}}{x}=\lim\limits_{x\to0}\frac{\frac{e^{2x}-1}{e^x}}{x}=\lim\limits_{x\to0}\frac{e^{2x}-1}{xe^x}=\lim\limits_{x\to0}\frac{e^{2x}-1}{x}\times\lim\limits_{x\to0}\frac{1}{e^x}=$

$$=2\lim_{x\to0}\frac{e^{2x}-1}{2x}\times\frac{1}{e^0}\underset{\substack{x\to0\\2x\to0}}{=}2\lim_{2x\to0}\frac{e^{2x}-1}{x}\times1=2\times1=2$$

2.6. $\lim\limits_{x\to0}\frac{a^x-1}{x}$, com $a\in IR^+\setminus\{1\}$

$$\lim_{x\to0}\frac{a^x-1}{x}=\lim_{x\to0}\frac{e^{\ln a^x}-1}{x}=\lim_{x\to0}\frac{e^{x\ln a}-1}{x}=\ln a\lim_{x\to0}\frac{e^{x\ln a}-1}{x\ln a}\underset{\substack{x\to0\\x\ln a\to0\\a\in IR^+\setminus\{1\}}}{=}\ln a\lim_{x\ln a\to0}\frac{e^{x\ln a}-1}{x\ln a}=\ln a\times1=\ln a$$

2.7. $\lim_{x\to0}\frac{\ln(x+1)}{x}\underset{\substack{y=\ln(x+1)\Leftrightarrow x+1=e^y\Leftrightarrow x=e^y-1\\ x\to0,y\to0}}{=}\lim_{y\to0}\frac{y}{e^y-1}=\frac{1}{\lim_{y\to0}\frac{e^y-1}{y}}=\frac{1}{1}=1$

2.8. $\lim_{x\to1}\frac{\ln x}{x-1}\underset{\substack{y=\ln x\Leftrightarrow x=e^y\\ x\to1,\ y\to0}}{=}\lim_{y\to0}\frac{y}{e^y-1}=\frac{1}{\lim_{y\to0}\frac{e^y-1}{y}}=\frac{1}{1}=1$

2.9. $\lim_{x\to+\infty}\frac{5x^5-e^x}{-x^{10}}=\lim_{x\to+\infty}\frac{5x^2}{-x^{10}}+\lim_{x\to+\infty}\frac{-e^x}{-x^{10}}=-5\lim_{x\to+\infty}\frac{x^2}{x^{10}}+\lim_{x\to+\infty}\frac{e^x}{x^{10}}=-5\times0+\infty=+\infty$

2.10. $\lim_{x\to3}\frac{e^x-e^3}{x-3}\underset{\substack{x\to3\Leftrightarrow x-3\to0\\ y=x-3\Leftrightarrow x=y+3\\ x\to3,\ y\to0}}{=}\lim_{y\to0}\frac{e^{y+3}-e^3}{y}=e^3\lim_{y\to0}\frac{e^y-1}{y}=e^3\times1=e^3$

2.11. $\lim_{x\to+\infty}\frac{e^{2x}-e^x}{x^e}=\lim_{x\to+\infty}\frac{e^x\left(e^x-1\right)}{x^e}=\lim_{x\to+\infty}\frac{e^x}{x}\times\lim_{x\to+\infty}\left(e^x-1\right)=+\infty\times(+\infty)=+\infty$

2.12. $\lim_{x\to+\infty}\frac{4^x-2^x}{x}=\lim_{x\to+\infty}\frac{\left(2^2\right)^x-2^x}{x}=\lim_{x\to+\infty}\frac{2^{2x}-2^x}{x}=\lim_{x\to+\infty}\frac{2^x\left(2^x-1\right)}{x}=\lim_{x\to+\infty}\frac{2^x}{x}\times\lim_{x\to+\infty}\left(2^x-1\right)=$

$=\lim_{x\to+\infty}\frac{e^{\ln 2^x}}{x}\times\lim_{x\to+\infty}\left(2^x-1\right)=\lim_{x\to+\infty}\frac{e^{x\ln2}}{x}\times\lim_{x\to+\infty}\left(2^x-1\right)=\lim_{x\to+\infty}\left(\frac{e^x}{x^{\frac{1}{\ln2}}}\right)^{\ln2}\times\lim_{x\to+\infty}\left(2^x-1\right)=$

$=(+\infty)^{\ln2}\times(+\infty)=+\infty$

2.13. $\lim_{x\to-\infty}\left(x^ke^x\right)$, com $k\in IN$

$\lim_{x\to-\infty}\left(x^ke^x\right)^{(\infty\times0)}\underset{\substack{x\to-\infty,\ -x\to+\infty\\ y=-x\Leftrightarrow x=-y\\ y\to+\infty}}{=}\lim_{y\to+\infty}\left((-y)^ke^{-y}\right)=(-1)^k\lim_{y\to+\infty}\frac{y^k}{e^y}=(-1)^k\times\frac{1}{\lim_{y\to+\infty}\frac{e^y}{y^k}}=(-1)^k\times\frac{1}{+\infty}=$

$=(-1)^k\times0=0$

2.14. $\lim_{x\to+\infty}\frac{3x^2+2+\ln x}{x}=\lim_{x\to+\infty}\frac{3x^2+2}{x}+\lim_{x\to+\infty}\frac{\ln x}{x}=\lim_{x\to+\infty}\frac{3x^2}{x}+\infty=+\infty+\infty=+\infty$

2.15. $\lim_{x\to+\infty}\frac{\ln\left(2x^4\right)}{3x}=\lim_{x\to+\infty}\frac{\ln2+\ln x^4}{3x}=\lim_{x\to+\infty}\frac{\ln2}{3x}+\lim_{x\to+\infty}\frac{4\ln x}{3x}=0+\frac{3}{4}\lim_{x\to+\infty}\frac{\ln x}{x}=0+\frac{3}{4}\times0=0$

2.16. $\lim\limits_{x\to+\infty}\dfrac{\log_a x}{x}$, com $a\in IR^+\setminus\{1\}$

$$\lim_{x\to+\infty}\frac{\log_a x}{x}=\lim_{x\to+\infty}\frac{\frac{\ln x}{\ln a}}{x}=\lim_{x\to+\infty}\frac{\ln x}{\ln a\times x}\underset{\substack{\ln a\neq 0\\ a\in IR^+\setminus\{1\}}}{=}\frac{1}{\ln a}\lim_{x\to+\infty}\frac{\ln x}{x}=\frac{1}{\ln a}\times 0=0$$

2.17. $$\lim_{x\to 0^+}(x\ln x)\underset{\substack{y=\frac{1}{x}\Leftrightarrow x=\frac{1}{y}\\ x\to 0^+,\ y\to\frac{1}{0^+},\ y\to+\infty}}{=}\lim_{y\to+\infty}\frac{1}{y}\ln\left(\frac{1}{y}\right)=\lim_{y\to+\infty}\frac{\ln y^{-1}}{y}=-\lim_{y\to+\infty}\frac{\ln y}{y}=-1\times 0=0$$

2.18. $$\lim_{x\to 1}\frac{e^x-e}{ex-e}\underset{\substack{x\to 1\\ x-1\to 0\\ y=x-1\Leftrightarrow x=y+1\\ y\to 0}}{=}\lim_{y\to 0}\frac{e^{y+1}-e}{e(y+1-1)}=\lim_{y\to 0}\frac{\cancel{e}\left(e^y-1\right)}{\cancel{e}y}=\lim_{y\to 0}\frac{e^y-1}{y}=1$$

2.19 $$\lim_{x\to 1}\frac{xe^x-e}{3x-3}\underset{\substack{x\to 1\\ x-1\to 0\\ y=x-1\Leftrightarrow x=y+1\\ y\to 0}}{=}\lim_{y\to 0}\frac{(y+1)e^{y+1}-e}{3y}=\lim_{y\to 0}\frac{(y+1)e\left(e^y-1\right)}{3y}=\lim_{y\to 0}\frac{e(y+1)}{3}\times\lim_{y\to 0}\frac{e^y-1}{y}=$$

$$=\frac{e}{3}\times 1=\frac{e}{3}$$

2.20. $$\lim_{x\to-\infty}\left(xe^x\right)\underset{\substack{x\to-\infty\\ -x\to+\infty\\ y=-x\Leftrightarrow x=-y\\ y\to+\infty}}{=}\lim_{y\to+\infty}\left(-ye^{-y}\right)=-\lim_{y\to+\infty}\frac{y}{e^y}=-\frac{1}{\lim\limits_{y\to+\infty}\frac{e^y}{y}}=-\frac{1}{+\infty}=0$$

2.21. $$\lim_{x\to+\infty}\frac{e^x-3^x}{x}=\lim_{x\to+\infty}\frac{e^x\left(1-\frac{3^x}{e^x}\right)}{x}=\lim_{x\to+\infty}\frac{e^x}{x}\times\lim_{x\to+\infty}\left(1-\left(\frac{3}{e}\right)^x\right)=+\infty\times(1-\infty)=-\infty$$

2.22. $\lim\limits_{x\to+\infty}\left[\ln\left(e^x-2x\right)-x\right]$

$$\lim_{x\to+\infty}\left[\ln\left(e^x-2x\right)-x\right]=\lim_{x\to+\infty}\left[\ln\left(e^x\left(1-\frac{2x}{e^x}\right)\right)-x\right]=\lim_{x\to+\infty}\left[\ln e^x+\ln\left(1-\frac{2x}{e^x}\right)-x\right]=$$

$$=\lim_{x\to+\infty}\left[x+\ln\left(1-\frac{2x}{e^x}\right)-x\right]=\lim_{x\to+\infty}\ln\left(1-\frac{2x}{e^x}\right)=\ln\left(\lim_{x\to+\infty}\left(1-\frac{2x}{e^x}\right)\right)=\ln\left(1-\lim_{x\to+\infty}\frac{2x}{e^x}\right)=$$

$$=\ln\left(1-\frac{2}{\lim\limits_{x\to+\infty}\frac{e^x}{x}}\right)=\ln\left(1-\frac{2}{+\infty}\right)=\ln(1-0)=\ln 1=0$$

2.23. $\lim_{x\to+\infty} \frac{5^x}{x^5}$

$$\lim_{x\to+\infty} \frac{5^x}{x^5} = \lim_{x\to+\infty} \frac{e^{\ln 5^x}}{x^5} = \lim_{x\to+\infty} \frac{e^{x\ln 5}}{x^5} = \lim_{x\to+\infty} \left(\frac{e^x}{x^{\frac{5}{\ln 5}}} \right)^{\ln 5} = (+\infty)^{\ln 5} = +\infty$$

2.24. $\lim_{x\to+\infty} \frac{\log_5 x}{x^5}$

$$\lim_{x\to+\infty} \frac{\log_5 x}{x^5} = \lim_{x\to+\infty} \frac{\frac{\ln x}{\ln 5}}{x^5} = \lim_{x\to+\infty} \frac{\ln x}{x^5 \times \ln 5} = \frac{1}{\ln 5} \times \lim_{x\to+\infty} \frac{\ln x}{x} \times \lim_{x\to+\infty} \frac{1}{x^4} = \frac{1}{\ln 5} \times 0 \times 0 = 0$$

2.25. $\lim_{x\to+\infty} \frac{-3x^3 - 1 + \ln x}{x}$

$$\lim_{x\to+\infty} \frac{-3x^3 - 1 + \ln x}{x} = -\lim_{x\to+\infty} \frac{3x^3}{x} - \lim_{x\to+\infty} \frac{1}{x} - \lim_{x\to+\infty} \frac{\ln x}{x} = -\infty - \frac{1}{+\infty} - 0 = -\infty - 0 - 0 = -\infty$$

2.26. $\lim_{x\to 1} \frac{5^x - 2^x}{x^2 + 2x + 3}$

$$\lim_{x\to 1} \frac{5^x - 2^x}{x^2 + 2x + 3} = \frac{5-2}{1+2+3} = \frac{1}{2}$$

2.27. $\lim_{x\to 0} \left[3^x (5x-2) + 8 \right]$

$$\lim_{x\to 0} \left[3^x (5x-2) + 8 \right] = 1(0-2) + 8 = 6$$

2.28. $\lim_{x\to 3} \left(2^{x^2-5} \right)$

$$\lim_{x\to 3} \left(2^{x^2-5} \right) = 2^{9-5} = 2^4 = 16$$

2.29. $\lim_{x\to+\infty} \frac{e^x + x}{3}$

$$\lim_{x\to+\infty} \frac{e^x + x}{3} = \frac{+\infty + \infty}{3} = +\infty$$

2.30. $\lim_{x\to-\infty} \frac{e^x + 5}{x}$

$$\lim_{x\to-\infty} \frac{e^x + 5}{x} = \frac{0+5}{-\infty} = 0$$

2.31. $\lim\limits_{x\to-\infty} \frac{2}{5^{-x}+1}$

$\lim\limits_{x\to-\infty} \frac{2}{5^{-x}+1} = \frac{2}{+\infty+1} = 0$

2.32. $\lim\limits_{x\to-\infty} \frac{20}{10^{x}+5}$

$\lim\limits_{x\to-\infty} \frac{20}{10^{x}+5} = \frac{20}{0+5} = 4$

2.33. $\lim\limits_{x\to+\infty} \frac{1}{7^{x}-15}$

$\lim\limits_{x\to+\infty} \frac{1}{7^{x}-15} = \frac{1}{+\infty} = 0$

2.34. $\lim\limits_{x\to-\infty} \frac{1}{8^{-x}-3}$

$\lim\limits_{x\to-\infty} \frac{1}{8^{-x}-3} = \frac{1}{+\infty-3} = 0$

2.35. $\lim\limits_{x\to+\infty} \left[(3-4x)\times 2^{x}\right]$

$\lim\limits_{x\to+\infty} \left[(3-4x)\times 2^{x}\right] = -\infty\times(+\infty) = -\infty$

2.36. $\lim\limits_{x\to-\infty} \frac{6+3^{x}}{4^{x}-3}$

$\lim\limits_{x\to-\infty} \frac{6+3^{x}}{4^{x}-3} = \frac{6+0}{+\infty-3} = 0$

2.37. $\lim\limits_{x\to+\infty} \left(2^{x^2-3x+5}\right)$

$\lim\limits_{x\to+\infty} \left(2^{x^2-3x+5}\right) = 2^{\lim\limits_{x\to+\infty}\left(x^2\left(1-\frac{3}{x}+\frac{5}{x^2}\right)\right)} = 2^{+\infty} = +\infty$

2.38. $\lim\limits_{x\to-\infty} \left(3^{-x^2+x+2}\right)$

$\lim\limits_{x\to-\infty} \left(3^{-x^2+x+2}\right) = 3^{\lim\limits_{x\to-\infty}\left(-x^2\left(1-\frac{1}{x}-\frac{2}{x^2}\right)\right)} = 3^{-(+\infty)\times\left(1-\frac{1}{-\infty}-\frac{2}{+\infty}\right)} = 3^{-\infty} = 0$

2.39. $\lim\limits_{x\to 1} \dfrac{4^x - 4}{2^x - 2}$

$$\lim_{x\to 1} \frac{4^x - 4}{2^x - 2}^{\left(\frac{0}{0}\right)} = \lim_{x\to 1} \frac{\left(2^x - 2\right)\left(2^x + 2\right)}{\left(2^x - 2\right)} = \lim_{x\to 1}\left(2^x + 2\right) = 4$$

2.40. $\lim\limits_{x\to +\infty} \dfrac{2\times 5^x}{3\times 10^x + 1}$

$$\lim_{x\to +\infty} \frac{2\times 5^x}{3\times 10^x + 1} = \lim_{x\to +\infty} \frac{2}{\frac{3\times 10^x}{5^x} + \frac{1}{5^x}} = \lim_{x\to +\infty} \frac{2}{3\times 2^x + \frac{1}{5^x}} = \frac{2}{+\infty + 0} = 0$$

2.41. $\lim\limits_{x\to +\infty} \left(7^x - 3^x\right)$

$$\lim_{x\to +\infty} \left(7^x - 3^x\right)^{(\infty-\infty)} = \lim_{x\to +\infty}\left(7^x\left(1 - \frac{3^x}{7^x}\right)\right) = \lim_{x\to +\infty}\left(7^x\left(1 - \left(\frac{3}{7}\right)^x\right)\right) = +\infty\left(1 - 0\right) = +\infty$$

2.42. $\lim\limits_{x\to -\infty} \dfrac{3^x - 2^x}{5^x}$

$$\lim_{x\to -\infty} \frac{3^x - 2^x}{5^x}^{\left(\frac{0}{0}\right)} = \lim_{x\to -\infty} \frac{3^x\left(1 - \left(\frac{2}{3}\right)^x\right)}{5^x} = \lim_{x\to -\infty}\left(\frac{3}{5}\right)^x \times \lim_{x\to -\infty} 1 - \left(\frac{2}{3}\right)^x = +\infty \times \left(1 - \left(+\infty\right)\right) = -\infty$$

2.43. $\lim\limits_{x\to +\infty} \dfrac{3^x + 2^x}{5^x}$

$$\lim_{x\to +\infty} \frac{3^x + 2^x}{5^x} = \lim_{x\to +\infty}\left(\frac{3}{5}\right)^x + \lim_{x\to +\infty}\left(\frac{2}{5}\right)^x = 0 + 0 = 0$$

2.44. $\lim\limits_{x\to +\infty} \dfrac{e^x}{x^{2013}}$

$$\lim_{x\to +\infty} \frac{e^x}{x^{2013}} = +\infty$$

2.45. $\lim\limits_{x\to -\infty} \dfrac{e^x}{x^{2013}}$

$$\lim_{x\to -\infty} \frac{e^x}{x^{2013}} = \frac{0}{-\infty} = 0$$

2.46. $\lim\limits_{x\to+\infty}\dfrac{x^{2013}}{e^x}$

$$\lim_{x\to+\infty}\frac{x^{2013}}{e^x}=\frac{1}{\lim\limits_{x\to+\infty}\dfrac{e^x}{x^{2013}}}=\frac{1}{+\infty}=0$$

2.47. $\lim\limits_{x\to+\infty}\dfrac{3x^5-2^x}{x^5}$

$$\lim_{x\to+\infty}\frac{3x^5-2^x}{x^5}=\lim_{x\to+\infty}\frac{3\cancel{x^5}}{\cancel{x^5}}-\lim_{x\to+\infty}\frac{2^x}{x^5}=3-\lim_{x\to+\infty}\frac{e^{\ln 2^x}}{x^5}=3-\lim_{x\to+\infty}\frac{e^{x\ln 2}}{x^5}=$$

$$=3-\lim_{x\to+\infty}\left(\frac{e^x}{x^{\frac{5}{\ln 2}}}\right)^{\ln 2}=3-(+\infty)=-\infty$$

2.48. $\lim\limits_{x\to+\infty}\dfrac{3^x-2^x}{x}$

$$\lim_{x\to+\infty}\frac{3^x-2^x}{x}=\lim_{x\to+\infty}\frac{3^x\left(1-\left(\frac{2}{3}\right)^x\right)}{x}=\lim_{x\to+\infty}\frac{3^x}{x}\times\lim_{x\to+\infty}\left(1-\left(\frac{2}{3}\right)^x\right)=$$

$$=\lim_{x\to+\infty}\frac{e^{\ln 3^x}}{x}\times(1-0)=\lim_{x\to+\infty}\frac{e^{x\ln 3}}{x}=\lim_{x\to+\infty}\left(\frac{e^x}{x^{\frac{1}{\ln 3}}}\right)^{\ln 3}=+\infty$$

2.49. $\lim\limits_{x\to+\infty}\dfrac{3^x-5^x}{x^2}$

$$\lim_{x\to+\infty}\frac{3^x-5^x}{x^2}=\lim_{x\to+\infty}\frac{5^x\left(\left(\frac{3}{5}\right)^x-1\right)}{x^2}=\lim_{x\to+\infty}\frac{5^x}{x^2}\times\lim_{x\to+\infty}\left(\left(\frac{3}{5}\right)^x-1\right)=$$

$$=\lim_{x\to+\infty}\frac{e^{\ln 5^x}}{x^2}\times(0-1)=-\lim_{x\to+\infty}\frac{e^{x\ln 5}}{x^2}=-\lim_{x\to+\infty}\left(\frac{e^x}{x^{\frac{2}{\ln 5}}}\right)^{\ln 5}=-(+\infty)=-\infty$$

2.50. $\lim\limits_{x\to+\infty}\dfrac{e^x+x+10}{x^2}$

$$\lim_{x\to+\infty}\frac{e^x+x+10}{x^2}=\lim_{x\to+\infty}\frac{e^x}{x^2}+\lim_{x\to+\infty}\frac{x}{x^2}+\lim_{x\to+\infty}\frac{10}{x^2}=+\infty+0+0=+\infty$$

2.51. $\lim\limits_{x\to-\infty}\dfrac{e^x+x+10}{x^2}$

$$\lim_{x\to-\infty}\frac{e^x+x+10}{x^2}=\lim_{x\to-\infty}\frac{e^x}{x^2}+\lim_{x\to-\infty}\frac{x}{x^2}+\lim_{x\to-\infty}\frac{10}{x^2}=\frac{0}{+\infty}+0+0=0$$

2.52. $\lim\limits_{x\to-\infty}\left(e^x x^4\right)$

$$\lim_{x\to-\infty}\left(e^x x^4\right)^{(0\times\infty)}=\lim_{y\to+\infty}\left(e^{-y}(-y)^4\right)=\lim_{y\to+\infty}\frac{y^4}{e^y}=\frac{1}{\lim\limits_{y\to+\infty}\frac{e^y}{y^4}}=\frac{1}{+\infty}=0$$

$x\to-\infty\,,\ -x\to+\infty$

$y=-x\Leftrightarrow x=-y$

$y\to+\infty$

2.53. $\lim\limits_{x\to0^+}\left(xe^{\frac{1}{x}}\right)$

$$\lim_{x\to0^+}\left(xe^{\frac{1}{x}}\right)^{(0\times\infty)}=\lim_{y\to+\infty}\left(\frac{1}{y}e^y\right)=\lim_{y\to+\infty}\frac{e^y}{y}=+\infty$$

$y=\dfrac{1}{x}\Leftrightarrow x=\dfrac{1}{y}$

$x\to0^+$

$y\to\dfrac{1}{0^+}\,,\ y\to+\infty$

2.54. $\lim\limits_{x\to0}\dfrac{1-e^{5x}}{x}$

$$\lim_{x\to0}\frac{1-e^{5x}}{x}^{\left(\frac{0}{0}\right)}=-5\lim_{x\to0}\frac{e^{5x}-1}{5x}=-5\lim_{5x\to0}\frac{e^{5x}-1}{5x}=-5\times1=-5$$

$x\to0$

$5x\to0$

2.55. $\lim\limits_{x\to0}\dfrac{4e^x-4}{3x}$

$$\lim_{x\to0}\frac{4e^x-4}{3x}=\frac{4}{3}\lim_{x\to0}\frac{e^x-1}{x}=\frac{4}{3}\times1=\frac{4}{3}$$

2.56. $\lim\limits_{x\to 0}\dfrac{e^{2x}-1}{5x}$

$$\lim_{x\to 0}\frac{e^{2x}-1}{5x}=\frac{2}{5}\lim_{x\to 0}\frac{e^{2x}-1}{2x}\underset{\substack{x\to 0\\2x\to 0}}{=}\frac{2}{5}\lim_{2x\to 0}\frac{e^{2x}-1}{2x}=\frac{2}{5}\times 1=\frac{2}{5}$$

2.57. $\lim\limits_{x\to 0}\dfrac{2x}{1-e^{4x}}$

$$\lim_{x\to 0}\frac{2x}{1-e^{4x}}=\frac{2}{-\lim\limits_{x\to 0}\dfrac{e^{4x}-1}{x}}=\frac{2}{-4\lim\limits_{x\to 0}\dfrac{e^{4x}-1}{4x}}\underset{\substack{x\to 0\\4x\to 0}}{=}\frac{2}{-4\lim\limits_{4x\to 0}\dfrac{e^{4x}-1}{4x}}=\frac{2}{-4\times 1}=-\frac{1}{2}$$

2.58. $\lim\limits_{x\to -2}\dfrac{e^{x+2}-1}{x^2-4}$

$$\lim_{x\to -2}\frac{e^{x+2}-1}{x^2-4}=\lim_{x\to -2}\frac{e^{x+2}-1}{(x+2)(x-2)}=\lim_{x\to -2}\frac{e^{x+2}-1}{x+2}\times\lim_{x\to -2}\frac{1}{x-2}\underset{\substack{x\to -2,\ x+2\to 0\\y=x+2,\ y\to 0}}{=}$$

$$=\lim_{y\to 0}\frac{e^{y}-1}{y}\times\frac{1}{-2-2}=1\times\left(-\frac{1}{4}\right)=-\frac{1}{4}$$

2.59. $\lim\limits_{x\to 0}\dfrac{e^{x}-1}{e^{5x}-1}$

$$\lim_{x\to 0}\frac{e^{x}-1}{e^{5x}-1}=\lim_{x\to 0}\frac{\dfrac{e^{x}-1}{x}}{\dfrac{e^{5x}-1}{x}}=\frac{\lim\limits_{x\to 0}\dfrac{e^{x}-1}{x}}{5\lim\limits_{x\to 0}\dfrac{e^{5x}-1}{5x}}\underset{\substack{x\to 0\\5x\to 0}}{=}\frac{1}{\lim\limits_{5x\to 0}\dfrac{e^{5x}-1}{5x}}=\frac{1}{1}=1$$

2.60. $\lim\limits_{x\to 0}\dfrac{e^{x+5}-e^{5}}{x}$

$$\lim_{x\to 0}\frac{e^{x+5}-e^{5}}{x}=\lim_{x\to 0}\frac{e^{5}\left(e^{x}-1\right)}{x}=e^{5}\lim_{x\to 0}\frac{e^{x}-1}{x}=e^{5}\times 1=e^{5}$$

2.61. $\lim\limits_{x\to 1}\dfrac{e^{x}-e}{x-1}$

$$\lim_{x\to 1}\frac{e^{x}-e}{x-1}\underset{\substack{x\to 1,\ x-1\to 0\\y=x-1,\ y\to 0}}{=}\lim_{y\to 0}\frac{e^{y+1}-e}{y}=e\lim_{y\to 0}\frac{e^{y}-1}{y}=e\times 1=e$$

2.62. $\lim\limits_{x\to 2} \dfrac{e^x - e^2}{x-2}$

$$\lim_{x\to 2} \frac{e^x - e^2}{x-2} \underset{\substack{x \to 2,\ x-2 \to 0 \\ y = x-2,\ y\to 0}}{=} \lim_{y\to 0} \frac{e^{y+2} - e^2}{y} = e^2 \lim_{y\to 0} \frac{e^y - 1}{y} = e^2 \times 1 = e^2$$

2.63. $\lim\limits_{x\to 0} \dfrac{e^x - e^{-x}}{2x}$

$$\lim_{x\to 0} \frac{e^x - e^{-x}}{2x} = \lim_{x\to 0} \frac{e^{-x}\left(e^{2x} - 1\right)}{2x} = \lim_{x\to 0} \frac{1}{e^x} \times \lim_{x\to 0} \frac{e^{2x} - 1}{2x} \underset{\substack{x\to 0 \\ 2x\to 0}}{=} \lim_{x\to 0} \frac{1}{e^x} \times \lim_{2x\to 0} \frac{e^{2x} - 1}{2x} =$$

$$= 1 \times 1 = 1$$

2.64. $\lim\limits_{x\to -\infty} \left[x\left(e^{\frac{1}{x}} - 1 \right) \right]$

$$\lim_{x\to -\infty} \left[x\left(e^{\frac{1}{x}} - 1 \right) \right]^{(\infty\times 0)} \underset{\substack{y=\frac{1}{x} \Leftrightarrow x=\frac{1}{y} \\ x\to -\infty,\ y\to 0}}{=} \lim_{y\to 0} \frac{e^y - 1}{x} = 1$$

2.65. $\lim\limits_{x\to 8} \left(2 - 3\log_2(x)\right)$

$$\lim_{x\to 8} \left(2 - 3\log_2(x)\right) = 2 - 3\log_2 8 = 2 - 3\times 3 = -7$$

2.66. $\lim\limits_{x\to 1} \dfrac{1 + \ln(x)}{x+2}$

$$\lim_{x\to 1} \frac{1 + \ln(x)}{x+2} = \frac{1 + \ln 1}{1+2} = \frac{1+0}{3} = \frac{1}{3}$$

2.67. $\lim\limits_{x\to 0^+} \left(\log_3(x)\right)^2$

$$\lim_{x\to 0^+} \left(\log_3(x)\right)^2 = \left(\log_3 0^+\right)^2 = \left(-\infty\right)^2 = +\infty$$

2.69. $\lim\limits_{x\to 0^+} \dfrac{\log x}{x^2 - x}$

$$\lim_{x\to 0^+} \frac{\log x}{x^2 - x} = \frac{-\infty}{0^+} = -\infty$$

2.102 $\lim\limits_{x\to 0}\dfrac{x}{\ln\left(x^2+2x+4\right)-\ln\left(x+4\right)}$

$$\lim_{x\to 0}\frac{x}{\ln\left(x^2+2x+4\right)-\ln\left(x+4\right)}=\lim_{x\to 0}\frac{x}{\ln\left(\dfrac{x^2+2x+4}{x+4}\right)}=\lim_{x\to 0}\frac{x}{\ln\left(\dfrac{x^2+x}{x+4}+1\right)}=$$

$$=\frac{1}{\lim\limits_{x\to 0}\left(\dfrac{\ln\left(\dfrac{x^2+x}{x+4}+1\right)}{x}\right)}=\frac{1}{\lim\limits_{x\to 0}\dfrac{\ln\left(\dfrac{x^2+x}{x+4}+1\right)}{\dfrac{x\left(x+1\right)}{x+4}}\times\dfrac{x+1}{x+4}}=\frac{1}{\lim\limits_{x\to 0}\dfrac{\ln\left(\dfrac{x^2+x}{x+4}+1\right)}{\dfrac{x^2+x}{x+4}}\times\lim\limits_{x\to 0}\dfrac{x+1}{x+4}}=$$

$$\underset{\substack{x\to 0\\ y=\frac{x^2+x}{x+4}\\ y\to 0}}{=}\frac{1}{\lim\limits_{y\to 0}\dfrac{\ln\left(y+1\right)}{y}\times\lim\limits_{x\to 0}\dfrac{x+1}{x+4}}\underset{\substack{y\to 0\\ z=\ln(y+1)\Leftrightarrow\\ \Leftrightarrow e^z-1=y\\ z\to 0}}{=}\frac{1}{\lim\limits_{z\to 0}\dfrac{z}{e^z-1}\times\lim\limits_{x\to 0}\dfrac{x+1}{x+4}}=\frac{\lim\limits_{z\to 0}\dfrac{e^z-1}{z}}{\lim\limits_{x\to 0}\dfrac{x+1}{x+4}}=\frac{1}{\dfrac{1}{4}}=4$$

2.135 $\lim\limits_{x\to -\infty}\dfrac{-\sqrt{x^2+x}}{\ln\left(-x\right)}$

$$\lim_{x\to -\infty}\frac{-\sqrt{x^2+x}}{\ln\left(-x\right)}=-\lim_{x\to -\infty}\frac{|x|\sqrt{1+\dfrac{1}{x}}}{\ln\left(-x\right)}=-\lim_{x\to -\infty}\frac{-x}{\ln\left(-x\right)}\times\lim_{x\to -\infty}\sqrt{1+\frac{1}{x}}\underset{\substack{x\to -\infty,\ -x\to +\infty\\ y=-x,\ y\to +\infty}}{=}$$

$$=-\lim_{y\to +\infty}\frac{y}{\ln\left(y\right)}\times\sqrt{1+0}=-\frac{1}{\lim\limits_{y\to +\infty}\dfrac{\ln y}{y}}=-\frac{1}{0}=-\infty$$

2.200 $\lim\limits_{x\to 2}\dfrac{\log_2\left(3x+2\right)-\log_2 x-2}{x-2}$

$$=\lim_{x\to 2}\frac{\log_2\left(\dfrac{3x+2}{x}\right)-2}{x-2}=\lim_{y\to 0}\frac{y}{\dfrac{2}{2^{y+2}-3}-2}=\lim_{y\to 0}\frac{y}{\dfrac{2-2\cdot 2^{y+2}+6}{2^{y+2}-3}}=\lim_{y\to 0}\frac{y\left(2^{y+2}-3\right)}{8-2\cdot 2^2\cdot 2^y}=$$

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$y=\log_2\left(\frac{3x+2}{x}\right)-2\Leftrightarrow\log_2\left(\frac{3x+2}{x}\right)=y+2\Leftrightarrow\frac{3x+2}{x}=2^{y+2}\Leftrightarrow 3x+2=x2^{y+2}\Leftrightarrow x2^{y+2}-3x=2\Leftrightarrow x\left(2^{y+2}-3\right)=2\Leftrightarrow x=\frac{2}{2^{y+2}-3}$

$x\to 2$
$x-2\to 0$
$y\to 0$

$$=\lim_{y\to 0}\frac{y\left(2^{y+2}-3\right)}{8-8\cdot 2^y}=\lim_{y\to 0}\frac{y\left(2^{y+2}-3\right)}{-8\left(2^y-1\right)}=\lim_{y\to 0}\frac{2^{y+2}-3}{-8}\cdot\lim_{y\to 0}\frac{y}{2^y-1}=-\frac{1}{8}\lim_{y\to 0}\frac{y}{e^{\ln 2^y}-1}=$$

$$=-\frac{1}{8}\lim_{y\to 0}\frac{y}{e^{y\ln 2}-1}=-\frac{1}{8}\cdot\frac{1}{\lim\limits_{y\to 0}\dfrac{e^{y\ln 2}-1}{y}}=-\frac{1}{8}\cdot\frac{1}{\ln 2\lim\limits_{y\to 0}\dfrac{e^{y\ln 2}-1}{y\ln 2}}\underset{\substack{y\to 0\\ y\ln 2\to 0}}{=}-\frac{1}{8}\cdot\frac{1}{\ln 2\cdot 1}=-\frac{1}{8\ln 2}$$

2.201 $\lim\limits_{x\to+\infty}\dfrac{x^2+2x}{e^{x+2}-e^2}$

$$\lim_{x\to+\infty}\frac{x^2+2x}{e^{x+2}-e^2}=\lim_{x\to+\infty}\frac{x^2\left(1+\frac{2}{x}\right)}{e^2\left(e^x-1\right)}=\lim_{x\to+\infty}\frac{1+\frac{2}{x}}{\frac{e^2\left(e^x-1\right)}{x^2}}=\frac{\lim\limits_{x\to+\infty}\left(1+\frac{2}{x}\right)}{e^2\lim\limits_{x\to+\infty}\frac{e^x-1}{x^2}}=$$

$$=\frac{1+\frac{2}{+\infty}}{e^2\left[\lim\limits_{x\to+\infty}\frac{e^x}{x^2}-\lim\limits_{x\to+\infty}\frac{1}{x^2}\right]}=\frac{1+0}{e^2\left(+\infty-\frac{1}{+\infty}\right)}=\frac{1}{e^2\left(+\infty-o\right)}=\frac{1}{+\infty}=0$$

3. Funções trigonométricas

3.1 $\lim\limits_{x\to0}\dfrac{\operatorname{sen}(2x)-4x}{x}$

$$\lim_{x\to0}\frac{\operatorname{sen}(2x)-4x}{x}^{\left(\frac{0}{0}\right)}=\lim_{x\to0}\frac{\operatorname{sen}2x}{x}-\lim_{x\to0}\frac{4\cancel{x}}{\cancel{x}}=2\lim_{x\to0}\frac{\operatorname{sen}2x}{2x}-4\underset{\substack{x\to0\\2x\to0}}{=}$$

$$=2\lim_{2x\to0}\frac{\operatorname{sen}2x}{2x}-4=2\times1-4=-2$$

3.2 $\lim\limits_{x\to+\infty}\dfrac{x\operatorname{sen}\left(\frac{1}{x}\right)}{2^{-x}+2}$

$$\lim_{x\to+\infty}\frac{x\operatorname{sen}\left(\frac{1}{x}\right)}{2^{-x}+2}\underset{\substack{x\to+\infty,\ \frac{1}{x}\to0\\ y=\frac{1}{x}\Leftrightarrow x=\frac{1}{x}\\ y\to0}}{=}\lim_{y\to0}\frac{\frac{1}{y}\operatorname{sen}(y)}{\frac{1}{2^{\frac{1}{y}}}+2}=\frac{\lim\limits_{y\to0}\frac{\operatorname{sen}y}{y}}{\lim\limits_{y\to0}\left(\frac{1}{2^{\frac{1}{y}}}+2\right)}=\frac{1}{\frac{1}{2^{\frac{1}{0}}}+2}=\frac{1}{\frac{1}{2^{+\infty}}+2}=$$

$$=\frac{1}{0+2}=\frac{1}{2}$$

3.3 $\lim\limits_{x\to0}\dfrac{\cos(2x)-\cos^2x}{x}$

$$\lim_{x\to0}\frac{\cos(2x)-\cos^2x}{x}^{\left(\frac{0}{0}\right)}=\lim_{x\to0}\frac{\cancel{\cos^2x}-\operatorname{sen}^2x-\cancel{\cos^2x}}{x}=-\lim_{x\to0}\frac{\operatorname{sen}(x)}{x}\times\lim_{x\to0}\operatorname{sen}x=$$

$$=-1\times0=0$$

3.4 $\lim\limits_{x\to\frac{\pi}{2}^+} \dfrac{\tan x}{\cos\left(x+\frac{\pi}{4}\right)}$

$$\lim_{x\to\frac{\pi}{2}^+} \frac{\tan x}{\cos\left(x+\frac{\pi}{4}\right)} = \frac{-\infty}{-\frac{\sqrt{2}}{2}} = +\infty$$

3.5 $\lim\limits_{x\to 0} \dfrac{2x^2+5\tan(6x)}{3x}$

$$\lim_{x\to 0} \frac{2x^2+5\tan(6x)}{3x} = \frac{2+5\times 0}{0} = +\infty$$

3.6 $\lim\limits_{x\to 0} \dfrac{\operatorname{sen}(3x)}{\tan(4x)}$

$$\lim_{x\to 0} \frac{\operatorname{sen}(3x)}{\tan(4x)}^{\left(\frac{0}{0}\right)} = \lim_{x\to 0} \frac{\operatorname{sen}(3x)}{\frac{\operatorname{sen}(4x)}{\cos(4x)}} = \lim_{x\to 0} \frac{\operatorname{sen}(3x)\cos(4x)}{\operatorname{sen}(4x)} ==$$

$$\lim_{x\to 0} \frac{\operatorname{sen}(3x)}{\operatorname{sen}(4x)} \times \underbrace{\lim_{x\to 0} \cos(4x)}_{\cos 0=1} = \lim_{x\to 0} \frac{3\cancel{x} \times \frac{\operatorname{sen}(3x)}{3x}}{4\cancel{x} \times \frac{\operatorname{sen}(4x)}{4x}} = \frac{3}{4} \times \frac{\lim\limits_{x\to 0} \frac{\operatorname{sen} 3x}{3x} \; \substack{x\to 0 \\ 3x\to 0}}{\lim\limits_{x\to 0} \frac{\operatorname{sen} 4x}{4x} \; \substack{x\to 0 \\ 4x\to 0}} =$$

$$= \frac{3}{4} \times \frac{\lim\limits_{3x\to 0} \frac{\operatorname{sen} 3x}{3x}}{\lim\limits_{4x\to 0} \frac{\operatorname{sen} 4x}{4x}} = \frac{3}{4} \times \frac{1}{1} = \frac{3}{4}$$

3.7 $\lim\limits_{x\to 0} \dfrac{\cos\left(2x-\frac{\pi}{2}\right)}{\operatorname{sen}(3-x)}$

$$\lim_{x\to 0} \frac{\cos\left(2x-\frac{\pi}{2}\right)}{\operatorname{sen}(3-x)} = \frac{\cos\left(-\frac{\pi}{2}\right)}{\operatorname{sen} 3} = \frac{0}{\operatorname{sen} 3} = 0$$

3.8 $\lim\limits_{x\to\frac{3\pi}{2}} \dfrac{2x-3\pi}{\cos(x-\pi)}$

$$\lim_{x\to\frac{3\pi}{2}} \frac{\overbrace{2x-3\pi}^{2\left(x-\frac{3\pi}{2}\right)}}{\cos(x-\pi)} \overset{\left(\frac{0}{0}\right)}{\underset{\substack{x\to\frac{3\pi}{2},\ x-\frac{3\pi}{2}\to 0 \\ y=x-\frac{3\pi}{2} \Leftrightarrow x=y+\frac{3\pi}{2},\ y\to 0}}{=}} \lim_{y\to 0} \frac{2y}{\cos\left(y+\frac{3\pi}{2}-\pi\right)} =$$

$$= \lim_{y\to 0} \frac{2y}{\cos\left(y-\frac{\pi}{2}\right)} = \lim_{y\to 0} \frac{2y}{\operatorname{sen} y} = \frac{2}{\lim\limits_{y\to 0} \frac{\operatorname{sen} y}{y}} = \frac{2}{1} = 2$$

3.9 $\lim\limits_{x\to 0} \dfrac{\operatorname{sen}(x-4\pi)+\cos\left(\frac{8x-\pi}{2}\right)}{4x}$

$$\lim_{x\to 0} \frac{\overbrace{\operatorname{sen}(x-4\pi)}^{\operatorname{sen} x}+\cos\left(\frac{8x-\pi}{2}\right)}{4x} \overset{\left(\frac{0}{0}\right)}{=} \lim_{x\to 0} \frac{\operatorname{sen}(x)}{4x} + \lim_{x\to 0} \frac{\overbrace{\cos\left(4x-\frac{\pi}{2}\right)}^{\operatorname{sen}(4x)}}{4x} =$$

$$= \frac{1}{4} \lim_{x\to 0} \frac{\operatorname{sen} x}{x} + \lim_{x\to 0} \frac{\operatorname{sen}(4x)}{4x} \underset{\substack{x\to 0 \\ 4x\to 0}}{=} \frac{1}{4} \times 1 + \lim_{4x\to 0} \frac{\operatorname{sen}(4x)}{4x} = \frac{1}{4} + 1 = \frac{5}{4}$$

3.10 $\lim\limits_{x\to\frac{\pi}{2}} \dfrac{\cos(5x)}{\operatorname{sen}\left(\frac{x}{6}+\frac{11}{12}\pi\right)}$

$$\lim_{x\to\frac{\pi}{2}} \frac{\cos(5x)}{\operatorname{sen}\left(\frac{x}{6}+\frac{11}{12}\pi\right)} \overset{\left(\frac{0}{0}\right)}{=} \underset{\substack{x\to\frac{\pi}{2},\ x-\frac{\pi}{2}\to 0\\ y=x-\frac{\pi}{2}\Leftrightarrow x=y+\frac{\pi}{2}\\ y\to 0}}{} \lim_{y\to 0} \frac{\cos\left(5y+\frac{5}{2}\pi\right)}{\operatorname{sen}\left(\frac{y+\frac{\pi}{2}}{6}+\frac{11}{12}\pi\right)} =$$

$$= \lim_{y\to 0} \frac{\overbrace{\cos\left(5y+\frac{\pi}{2}+2\pi\right)}^{-\operatorname{sen}(5y)}}{\operatorname{sen}\left(\frac{\frac{2y+\pi}{2}}{6}+\frac{11}{2}\pi\right)} = \lim_{y\to 0} \frac{-\operatorname{sen}(5y)}{\operatorname{sen}\left(\frac{2y+\pi}{12}+\frac{11}{12}\pi\right)} = \lim_{y\to 0} \frac{-\operatorname{sen}(5y)}{\operatorname{sen}\left(\frac{y}{6}+\frac{\pi}{12}+\frac{11}{12}\pi\right)} =$$

$$= -\lim_{y\to 0} \frac{\operatorname{sen}(5y)}{\underbrace{\operatorname{sen}\left(\frac{y}{6}+\pi\right)}_{-\operatorname{sen}\left(\frac{y}{6}\right)}} = -\lim_{y\to 0} \frac{\operatorname{sen}(5y)}{-\operatorname{sen}\left(\frac{y}{6}\right)} = \lim_{y\to 0} \frac{5\cancel{y} \times \frac{\operatorname{sen}(5y)}{5y}}{\frac{\cancel{y}}{6} \times \frac{\operatorname{sen}\left(\frac{y}{6}\right)}{\frac{y}{6}}} =$$

$$= 30 \times \frac{\lim\limits_{y\to 0} \frac{\operatorname{sen}(5y)}{5y}}{\lim\limits_{y\to 0} \frac{\operatorname{sen}\left(\frac{y}{6}\right)}{\frac{y}{6}}} \underset{\substack{y\to 0\\ \frac{y}{6}\to 0}}{\overset{\substack{y\to 0\\ 5y\to 0}}{=}} 30 \times \frac{\lim\limits_{5y\to 0} \frac{\operatorname{sen}(5y)}{5y}}{\lim\limits_{\frac{y}{6}\to 0} \frac{\operatorname{sen}\left(\frac{y}{6}\right)}{\frac{y}{6}}} = 30 \times \frac{1}{1} = 30$$

3.11 $\lim\limits_{x\to 0} \dfrac{x^2}{2\cos(2x)-2}$

$$\lim_{x\to 0} \frac{x^2}{2\cos(2x)-2} = \lim_{x\to 0} \frac{x^2}{2\left(\cos^2 x-\operatorname{sen}^2 x\right)-2} = \frac{1}{2}\lim_{x\to 0} \frac{x^2}{\cos^2 x-1-\operatorname{sen}^2 x} =$$

$$= \frac{1}{2}\lim_{x\to 0} \frac{x^2}{-\left(1-\cos^2 x\right)-\operatorname{sen}^2 x} = \frac{1}{2}\lim_{x\to 0} \frac{x^2}{-\operatorname{sen}^2 x-\operatorname{sen}^2 x} = -\frac{1}{2}\lim_{x\to 0} \frac{x^2}{2\operatorname{sen}^2 x} =$$

$$= -\frac{1}{4}\left(\lim_{x\to 0} \frac{x}{\operatorname{sen} x}\right)^2 = -\frac{1}{4}\left(\frac{1}{\lim\limits_{x\to 0} \frac{\operatorname{sen} x}{x}}\right)^2 = -\frac{1}{4}\left(\frac{1}{1}\right)^2 = -\frac{1}{4}$$

3.12 $\lim\limits_{x\to -1} \dfrac{3x^2+9x+6}{\operatorname{sen}(x+1)}$

$$\lim_{x\to -1} \frac{3x^2+9x+6}{\operatorname{sen}(x+1)} = \lim_{x\to -1} \frac{(x+2)(x+1)}{\operatorname{sen}(x+1)} \underset{\substack{x\to -1,\ x+1\to 0 \\ y=x+1 \Leftrightarrow x=y-1 \\ y\to 0}}{=} \lim_{y\to 0} \frac{(y-1+2)y}{\operatorname{sen} y} =$$

$$= \lim_{y\to 0}(y+1) \times \lim_{y\to 0} \frac{y}{\operatorname{sen} y} = (0+1) \times \frac{1}{\lim\limits_{y\to 0} \dfrac{\operatorname{sen} y}{y}} = \frac{1}{1} = 1$$

3.13 $\lim\limits_{x\to 0^+} (\operatorname{sen} x)^{x^2}$

$$\lim_{x\to 0^+} (\operatorname{sen} x)^{x^2} \overset{(0^0)}{=} e^{\lim\limits_{x\to 0^+} \ln(\operatorname{sen} x)^{x^2}} = e^{\lim\limits_{x\to 0^+} x^2 \ln(\operatorname{sen} x)} = e^{\lim\limits_{x\to 0^+} \frac{x^2}{\operatorname{sen} x} \times \operatorname{sen} x \times \ln(\operatorname{sen} x)} =$$

$$= e^{\frac{\lim\limits_{x\to 0^+} x}{\lim\limits_{x\to 0^+} \frac{\operatorname{sen} x}{x}} \times \lim\limits_{x\to 0^+} \operatorname{sen} x \ln(\operatorname{sen} x)} \underset{\substack{x\to 0^+ \\ \frac{1}{x}\to +\infty \\ y=\frac{1}{\operatorname{sen} x} \Leftrightarrow \operatorname{sen} x=\frac{1}{y}}}{=} e^{\frac{\lim\limits_{x\to 0^+} x}{\lim\limits_{x\to 0^+} \frac{\operatorname{sen} x}{x}} \times \lim\limits_{y\to +\infty} \left(\frac{1}{y}\ln\left(\frac{1}{y}\right)\right)} = e^{\frac{\lim\limits_{x\to 0^+} x}{\lim\limits_{x\to 0^+} \frac{\operatorname{sen} x}{x}} \times \lim\limits_{y\to +\infty} \left(\frac{\ln y^{-1}}{y}\right)} =$$

$$= e^{\frac{\lim\limits_{x\to 0^+} x}{\lim\limits_{x\to 0^+} \frac{\operatorname{sen} x}{x}} \times \left(-\lim\limits_{y\to +\infty} \left(\frac{\ln y}{y}\right)\right)} = e^{\frac{0}{1} \times (-0)} = e^0 = 1$$

3.14 $\lim\limits_{x\to+\infty}\left(\cos\frac{4}{x}\right)^{x^2}$

$$\lim_{x\to+\infty}\left(\cos\frac{4}{x}\right)^{x^2}\ \left(0^{\infty}\right) = \lim_{x\to+\infty} e^{\ln\left(\cos\frac{4}{x}\right)^{x^2}} = e^{\lim\limits_{x\to+\infty}\ln\left(\cos\frac{4}{x}\right)^{x^2}} = e^{-8}$$

C.A.

$$\lim_{x\to+\infty}\ln\left(\cos\frac{4}{x}\right)^{x^2} = \lim_{x\to+\infty} x^2\ln\left(\cos\frac{4}{x}\right) =$$

$y=\ln\left(\frac{4}{x}\right)\Leftrightarrow e^y=\cos\frac{4}{x}\Leftrightarrow e^y-1=\cos\left(\frac{4}{x}\right)-1$

$x\to+\infty,\ y\to 0$

$$= \lim_{x\to+\infty} x^2\left(\cos\frac{4}{x}-1\right)\left(\frac{1}{\cos\frac{4}{x}-1}\right)\ln\left(\cos\frac{4}{x}\right) = \lim_{x\to+\infty} x^2\left(\cos\frac{4}{x}-1\right)\times\lim_{y\to0}\left(\frac{1}{e^y-1}\right)y =$$

$$= \lim_{x\to+\infty} x^2\left(\cos\frac{4}{x}-1\right)\times\lim_{y\to0}\left(\frac{y}{e^y-1}\right) = \lim_{x\to+\infty} x^2\left(\cos\frac{4}{x}-1\right)\times\left(\frac{1}{\lim\limits_{x\to0}\frac{e^y-1}{y}}\right) =$$

$$= \lim_{x\to+\infty} x^2\left(\cos\frac{4}{x}-1\right)\times\left(\frac{1}{1}\right) = \lim_{z\to0}\left(\frac{4}{z}\right)^2\left(\cos z-1\right)\times1 =$$

$z=\frac{4}{x}\Leftrightarrow x=\frac{4}{z}$

$x\to+\infty,\ z\to0$

$$= \lim_{z\to0}\frac{16\left(\cos z-1\right)\left(\cos z+1\right)}{z^2\left(\cos z+1\right)} = 16\lim_{z\to0}\frac{\left(\cos^2 z-1\right)}{z^2\left(\cos z+1\right)} = 16\lim_{z\to0}\frac{-\left(1-\cos^2 z\right)}{z^2\left(\cos z+1\right)} =$$

$$= -16\lim_{z\to0}\frac{\operatorname{sen}^2 z}{z^2\left(\cos z+1\right)} = -16\left[\lim_{z\to0}\left(\frac{\operatorname{sen} z}{z}\right)^2\times\lim_{z\to0}\frac{1}{\cos z+1}\right] =$$

$$= -16\left[\left(\lim_{z\to0}\frac{\operatorname{sen} z}{z}\right)^2\times\frac{1}{1+1}\right] = -16\left[\left(1\right)^1\times\frac{1}{2}\right] = -\frac{16}{2} = -8$$

3.15 $\lim\limits_{x\to0}\left(\frac{\cos^2 x-1}{x\operatorname{sen} x}\right)$

$$\lim_{x\to0}\left(\frac{\cos^2 x-1}{x\operatorname{sen} x}\right)^{\left(\frac{0}{0}\right)} = \lim_{x\to0}\frac{-\left(1-\cos^2 x\right)}{x\operatorname{sen} x} = -\lim_{x\to0}\frac{\operatorname{sen}^{2} x}{x\operatorname{sen} x} = -\lim_{x\to0}\frac{\operatorname{sen} x}{x} = -1$$

3.16 $\lim\limits_{x\to-\infty}\left(\frac{2x-\operatorname{sen} x}{x}\right)$

$$\lim_{x\to-\infty}\left(\frac{2x-\operatorname{sen} x}{x}\right) = \lim_{x\to-\infty}\frac{2x}{x} - \lim_{x\to-\infty}\frac{\operatorname{sen} x}{x} = 2-\frac{k}{-\infty} = 2-0 = 2$$

Como $-1\le\operatorname{sen} x\le1$

Seja $k\in\left[-1,1\right]$

3.17 $\lim_{x\to 0}\left(\frac{2x+\tan x}{\operatorname{sen} x}\right)$

$$\lim_{x\to 0}\left(\frac{2x+\tan x}{\operatorname{sen} x}\right)^{\left(\frac{0}{0}\right)} = \lim_{x\to 0}\frac{2x}{\operatorname{sen} x} + \lim_{x\to 0}\frac{\tan x}{\operatorname{sen} x} = \frac{2}{\lim_{x\to 0}\frac{\operatorname{sen} x}{x}} + \lim_{x\to 0}\frac{\frac{\operatorname{sen} x}{\cos x}}{\operatorname{sen} x} =$$

$$= \frac{2}{1} + \lim_{x\to 0}\frac{\cancel{\operatorname{sen} x}}{\cos x\,\cancel{\operatorname{sen} x}} = 2 + \frac{1}{1} = 3$$

3.18 $\lim_{x\to 0}\left(\frac{1-\cos x}{\operatorname{sen}^2 x}\right)$

$$\lim_{x\to 0}\left(\frac{1-\cos x}{\operatorname{sen}^2 x}\right)^{\left(\frac{0}{0}\right)} = \lim_{x\to 0}\frac{(1-\cos x)(1+\cos x)}{\operatorname{sen}^2 x\,(1+\cos x)} = \lim_{x\to 0}\frac{1-\cos^2 x}{\operatorname{sen}^2 x\,(1+\cos x)} =$$

$$= \lim_{x\to 0}\frac{\cancel{\operatorname{sen}^2 x}}{\cancel{\operatorname{sen}^2 x}\,(1+\cos x)} = \frac{1}{1+1} = \frac{1}{2}$$

3.19 $\lim_{x\to 0}\frac{(1+\cos x)\operatorname{sen}^2 x}{3x^2}$

$$\lim_{x\to 0}\frac{(1+\cos x)\operatorname{sen}^2 x}{3x^2}^{\left(\frac{0}{0}\right)} = \lim_{x\to 0}\frac{\operatorname{sen}^2 x}{x^2}\times\lim_{x\to 0}\frac{1+\cos x}{3} = \lim_{x\to 0}\left(\frac{\operatorname{sen} x}{x}\right)^2\times\frac{2}{3} =$$

$$= 1^2\times\frac{2}{3} = \frac{2}{3}$$

3.20 $\lim_{x\to 0}\frac{\operatorname{sen}(3x)}{\tan(2x)}$

$$\lim_{x\to 0}\frac{\operatorname{sen}(3x)}{\tan(2x)}^{\left(\frac{0}{0}\right)} = \lim_{x\to 0}\frac{\operatorname{sen}(3x)}{\frac{\operatorname{sen}(2x)}{\cos(2x)}} = \lim_{x\to 0}\frac{\operatorname{sen}(3x)}{\operatorname{sen}(2x)}\times\underbrace{\lim_{x\to 0}\cos(2x)}_{1} = \lim_{x\to 0}\frac{3\cancel{x}\times\frac{\operatorname{sen}(3x)}{3x}}{2\cancel{x}\,\frac{\operatorname{sen}(2x)}{2x}} =$$

$$= \frac{3}{2}\times\frac{\lim_{x\to 0}\frac{\operatorname{sen}(3x)}{3x}\ \substack{x\to 0\\3x\to 0}}{\lim_{x\to 0}\frac{\operatorname{sen}(2x)}{2x}\ \substack{x\to 0\\2x\to 0}} = \frac{3}{2}\times\frac{\lim_{3x\to 0}\frac{\operatorname{sen}(3x)}{3x}}{\lim_{2x\to 0}\frac{\operatorname{sen}(2x)}{2x}} = \frac{3}{2}\times\frac{1}{1} = \frac{3}{2}$$

3.21 $\lim_{x\to\frac{\pi}{4}} \frac{\tan x - 1}{4x - \pi}$

$$\lim_{x\to\frac{\pi}{4}} \frac{\tan x - 1}{4x-\pi}\left(\frac{0}{0}\right) = \lim_{x\to\frac{\pi}{4}} \frac{\frac{\text{sen } x}{\cos x} - 1}{4\left(x - \frac{\pi}{4}\right)} = \frac{1}{4}\lim_{x\to\frac{\pi}{4}} \frac{\frac{\text{sen } x - \cos x}{\cos x}}{x - \frac{\pi}{4}} = \frac{1}{4}\lim_{x\to\frac{\pi}{4}} \frac{\text{sen } x - \cos x}{\cos x\left(x - \frac{\pi}{4}\right)} =$$

$$= \frac{1}{4}\underbrace{\lim_{x\to\frac{\pi}{4}} \frac{1}{\cos x}}_{\cos\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}} \times \lim_{x\to\frac{\pi}{4}} \frac{\text{sen } x - \cos x}{x - \frac{\pi}{4}} = \frac{1}{4\times\frac{\sqrt{2}}{2}} \times \lim_{x\to\frac{\pi}{4}} \frac{\frac{2}{\sqrt{2}}\left(\text{sen } x \times \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\cos x\right)}{x - \frac{\pi}{4}} =$$

$$= \frac{\sqrt{2}}{4} \times \frac{2}{\sqrt{2}} \lim_{x\to\frac{\pi}{4}} \frac{\text{sen } x \cos\frac{\pi}{4} - \text{sen}\frac{\pi}{4}\cos x}{x - \frac{\pi}{4}} = \frac{2}{4}\times \lim_{x\to\frac{\pi}{4}} \frac{\text{sen}\left(x - \frac{\pi}{4}\right)}{x - \frac{\pi}{4}} \underset{\substack{x\to\frac{\pi}{4},\, x-\frac{\pi}{4}\to 0 \\ y = x - \frac{\pi}{4},\, y\to 0}}{=}$$

$$= \frac{1}{2}\lim_{y\to 0} \frac{\text{sen } y}{y} = \frac{1}{2}\times 1 = \frac{1}{2}$$

3.22 $\lim_{x\to 0} \frac{x^2 - x}{\text{sen } x}$

$$\lim_{x\to 0} \frac{x^2 - x}{\text{sen } x}\left(\frac{0}{0}\right) = \lim_{x\to 0} \frac{x(x-1)}{\text{sen } x} = \lim_{x\to 0}\frac{x}{\text{sen } x} \times \lim_{x\to 0}(x-1) = \frac{1}{\lim_{x\to 0}\frac{\text{sen } x}{x}} \times (0-1) =$$

$$= \frac{1}{1}\times(0-1) = -1$$

3.23 $\lim_{x\to 1} \frac{\pi\,\text{sen}(ax - a)}{x^2 - 1}$

$$\lim_{x\to 1} \frac{\pi\,\text{sen}(ax-a)}{x^2-1}\left(\frac{0}{0}\right) = \lim_{x\to 1}\frac{\pi\,\text{sen}(a(x-1))}{(x-1)(x+1)} =$$

$$= \underbrace{\lim_{x\to 1}\frac{\pi}{x+1}}_{\frac{\pi}{2}} \times \lim_{x\to 1}\frac{\text{sen}(a(x-1))}{x-1} \underset{\substack{x\to 1,\, x-1\to 0 \\ y = x-1,\, y\to 0}}{=} \frac{\pi}{2}\lim_{y\to 0}\frac{\text{sen}(ay)}{y} \underset{\substack{y\to 0 \\ ay\to 0}}{=}$$

$$= \frac{\pi}{2}\times a\lim_{x\to 1}\frac{\text{sen}(ay)}{ay} = \frac{a\pi}{2}$$

3.24 $\lim\limits_{n\to+\infty}\left[(2n+1)\,\text{sen}\,\frac{2}{n}\right]$

$$\lim_{n\to+\infty}\left[(2n+1)\,\text{sen}\,\frac{2}{n}\right]^{(0\times\infty)} \underset{\substack{n\to+\infty,\ \frac{2}{n}\to 0\\ y=\frac{2}{n}\Leftrightarrow n=\frac{2}{y}\\ y\to 0}}{=} \lim_{y\to 0}\left[\left(2\times\frac{2}{y}+1\right)\text{sen}\,y\right] = \lim_{y\to 0}\left[\left(\frac{4}{y}+1\right)\text{sen}\,y\right] =$$

$$= \lim_{y\to 0}\left(\frac{4\,\text{sen}\,y}{y}+\text{sen}\,y\right) = 4\lim_{y\to 0}\frac{\text{sen}\,y}{y} - \lim_{y\to 0}\text{sen}\,y = 4\times 1 - 0 = 4$$

3.25 $\lim\limits_{x\to 0}\frac{2x}{\tan(\pi x)}$

$$\lim_{x\to 0}\frac{2x}{\tan(\pi x)}^{\left(\frac{0}{0}\right)} = \lim_{x\to 0}\frac{2x}{\frac{\text{sen}(\pi x)}{\cos(\pi x)}} = 2\lim_{x\to 0}\frac{x\cos(\pi x)}{\text{sen}(\pi x)} = 2\underbrace{\lim_{x\to 0}\cos(\pi x)}_{1}\times\lim_{x\to 0}\frac{x}{\text{sen}(\pi x)} =$$

$$= \frac{2}{\pi}\lim_{x\to 0}\frac{\pi x}{\text{sen}(\pi x)} = \frac{2}{\pi}\times\frac{1}{\lim\limits_{x\to 0}\frac{\text{sen}(\pi x)}{\pi x}} \underset{\substack{x\to 0\\ \pi x\to 0}}{=} \frac{2}{\pi}\times\frac{1}{\lim\limits_{\pi x\to 0}\frac{\text{sen}(\pi x)}{\pi x}} = \frac{2}{\pi}\times\frac{1}{1} = \frac{2}{\pi}$$

3.26 $\lim\limits_{x\to\frac{\pi}{3}}\frac{\sqrt{3}\cos x - \text{sen}\,x}{3x-\pi}$

$$\lim_{x\to\frac{\pi}{3}}\frac{\sqrt{3}\cos x - \text{sen}\,x}{3x-\pi}^{\left(\frac{0}{0}\right)} = \lim_{x\to\frac{\pi}{3}}\frac{2\left(\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\text{sen}\,x\right)}{3\left(x-\frac{\pi}{3}\right)} = \frac{2}{3}\lim_{x\to\frac{\pi}{3}}\frac{\text{sen}\left(\frac{\pi}{3}-x\right)}{x-\frac{\pi}{3}} =$$

$$= \frac{2}{3}\lim_{x\to\frac{\pi}{3}}\frac{-\text{sen}\left(x-\frac{\pi}{3}\right)}{x-\frac{\pi}{3}} \underset{\substack{x\to\frac{\pi}{3},\ x-\frac{\pi}{3}\to 0\\ y=x-\frac{\pi}{3}}}{=} -\frac{2}{3}\lim_{y\to 0}\frac{\text{sen}\,y}{y} = -\frac{2}{3}\times 1 = -\frac{2}{3}$$

3.27 $\lim\limits_{x\to 0} \frac{x - \text{sen}\, x}{\sqrt{1-\cos x}}$

$$\lim_{x\to 0} \frac{x - \text{sen}\, x}{\sqrt{1-\cos x}} \overset{\left(\frac{0}{0}\right)}{=} \lim_{x\to 0} \frac{(x - \text{sen}\, x)\sqrt{1+\cos x}}{\sqrt{1-\cos x}\sqrt{1+\cos x}} =$$

$$= \lim_{x\to 0} \sqrt{1+\cos x} \times \lim_{x\to 0} \frac{x - \text{sen}\, x}{\sqrt{(1-\cos x)(1+\cos x)}} = \sqrt{2} \times \lim_{x\to 0} \frac{x - \text{sen}\, x}{\sqrt{(1-\cos^2 x)}} =$$

$$= \sqrt{2} \lim_{x\to 0} \frac{x - \text{sen}\, x}{\sqrt{\text{sen}^2\, x}} = \sqrt{2} \lim_{x\to 0} \frac{x - \text{sen}\, x}{|\text{sen}\, x|} = \sqrt{2} \lim_{x\to 0} \frac{\frac{x - \text{sen}\, x}{x}}{\frac{|\text{sen}\, x|}{x}} = \sqrt{2} \lim_{x\to 0} \frac{1 - \frac{\text{sen}\, x}{x}}{\frac{|\text{sen}\, x|}{x}} =$$

$$= \sqrt{2} \times \frac{1 - \lim\limits_{x\to 0} \frac{\text{sen}\, x}{x}}{\underbrace{\lim\limits_{x\to 0} \frac{|\text{sen}\, x|}{x}}} = \sqrt{2} \times 0 = 0$$

$$\lim_{x\to 0^+} \frac{\text{sen}\, x}{x} = 1 \Rightarrow \frac{1-1}{1} = 0$$

$$\lim_{x\to 0^-} \frac{-\text{sen}\, x}{x} = -1 \Rightarrow \frac{1-1}{-1} = 0$$

3.28 $\lim\limits_{x\to 0} \frac{1 - \text{sen}^2(2x) - \cos x}{x^2}$

$$\lim_{x\to 0} \frac{1 - \text{sen}^2(2x) - \cos x}{x^2} \overset{\left(\frac{0}{0}\right)}{=} \lim_{x\to 0} \frac{1 - \cos x - \text{sen}^2(2x)}{x^2} =$$

$$= \lim_{x\to 0} \frac{1-\cos x}{x^2} - \lim_{x\to 0} \frac{\text{sen}^2(2x)}{x^2}) = \lim_{x\to 0} \frac{(1-\cos x)(1+\cos x)}{x^2(1+\cos x)} - \left(\lim_{x\to 0} \frac{\text{sen}(2x)}{x}\right)^2 =$$

$$= \lim_{x\to 0} \frac{\overbrace{1-\cos^2 x}^{\text{sen}^2 x}}{x^2(1+\cos x)} - \left(2 \lim_{x\to 0} \frac{\text{sen}(2x)}{2x}\right)^2 = \quad \begin{matrix} x\to 0 \\ 2x\to 0 \end{matrix}$$

$$= \lim_{x\to 0} \frac{1}{1+\cos x} \times \lim_{x\to 0} \frac{\text{sen}^2 x}{x^2} - \left(2 \lim_{2x\to 0} \frac{\text{sen}(2x)}{2x}\right)^2 = \frac{1}{2}\left(\lim_{x\to 0} \frac{\text{sen}\, x}{x}\right)^2 - (2\times 1)^2 =$$

$$= \frac{1}{2}(1)^2 - 4 = \frac{1}{2} - 4 = -\frac{7}{2}$$

3.29 $\lim\limits_{x\to 0} \frac{\text{sen}\, x - \cos x + 1}{x}$

$$= \lim_{x\to 0} \frac{\text{sen}\, x}{x} - \lim_{x\to 0} \frac{\cos x - 1}{x} \overset{\left(\frac{0}{0}\right)}{=} 1 - \lim_{x\to 0} \frac{(\cos x - 1)(\cos x + 1)}{x(\cos x + 1)} = 1 + \lim_{x\to 0} \frac{\cos^2 x - 1}{x(\cos x + 1)} =$$

$$= 1 + \lim_{x\to 0} \frac{\text{sen}^2\, x}{x(\cos x + 1)} = 1 + \lim_{x\to 0} \frac{\text{sen}\, x}{x} \times \lim_{x\to 0} \frac{\text{sen}\, x}{\cos x + 1} = 1 + 1 \times \frac{0}{1+1} = 1$$

3.30 $\lim\limits_{x\to 0}\dfrac{5x}{\tan(-2x)}$

$$\lim_{x\to 0}\frac{5x}{\tan(-2x)}=5\lim_{x\to 0}\frac{x}{-\tan(2x)}=-5\lim_{x\to 0}\frac{x}{\tan(2x)}=-5\lim_{x\to 0}\frac{x}{\frac{\operatorname{sen}2x}{\cos 2x}}=$$

$$=-5\lim_{x\to 0}\frac{\cos 2x}{\frac{\operatorname{sen}2x}{x}}=-5\frac{\lim\limits_{x\to 0}\cos 2x}{2\lim\limits_{x\to 0}\frac{\operatorname{sen}2x}{2x}}\underset{\substack{x\to 0\\2x\to 0}}{}=-\frac{5}{2}\times\frac{1}{\lim\limits_{2x\to 0}\frac{\operatorname{sen}2x}{2x}}=-\frac{5}{2}\times\frac{1}{1}=-\frac{5}{2}$$

3.31 $\lim\limits_{x\to+\infty}\dfrac{2+\operatorname{sen}7x}{2x^2}$

$$\lim_{x\to+\infty}\frac{2+\operatorname{sen}7x}{2x^2}=\frac{2+k}{+\infty}=0$$

$f(x)=2+\operatorname{sen}7x$
é uma função limitada
$\forall x\in IR,\ 1\leq 2+\operatorname{sen}(7x)\leq 3$
Seja $k\in[1,3]$

3.32 $\lim\limits_{x\to 0}\dfrac{\operatorname{sen}(2x^3)}{\operatorname{sen}x(1-\cos^2 x)}$

$$\lim_{x\to 0}\frac{\operatorname{sen}(2x^3)}{\operatorname{sen}x(1-\cos^2 x)}\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{\operatorname{sen}(2x^3)}{\operatorname{sen}x\times\operatorname{sen}^2 x}\lim_{x\to 0}\frac{\operatorname{sen}(2x^3)}{\operatorname{sen}^3 x}=$$

$$=\frac{2\lim\limits_{x\to 0}\frac{\operatorname{sen}(2x^3)}{2x^3}}{\lim\limits_{x\to 0}\frac{\operatorname{sen}^3 x}{x^3}}\underset{x\to 0\Rightarrow 2x^3\to 0}{}=\frac{2\times 1}{\lim\limits_{x\to 0}\left(\frac{\operatorname{sen}x}{x}\right)^3}=\frac{2}{1^3}=2$$

3.33 $\lim\limits_{x\to 0}\dfrac{2x+\operatorname{sen}(2x)}{x+3x^3}$

$$\lim_{x\to 0}\frac{2x+\operatorname{sen}(2x)}{x+3x^3}\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{\cancel{x}\left(2+\frac{\operatorname{sen}(2x)}{x}\right)}{\cancel{x}(1+3x^2)}=\frac{\lim\limits_{x\to 0}\left(2+\frac{\operatorname{sen}(2x)}{x}\right)}{\lim\limits_{x\to 0}(1+3x^2)}=$$

$$=\frac{2+2\lim\limits_{x\to 0}\frac{\operatorname{sen}(2x)}{2x}}{1+0}=_{x\to 0\Rightarrow 2x\to 0}2+2\lim_{2x\to 0}\frac{\operatorname{sen}(2x)}{2x}=2+2\times 1=4$$

3.35 $\lim\limits_{x\to 0}\dfrac{x\operatorname{sen}x}{1-\cos x}$

$$\lim_{x\to 0}\frac{x\operatorname{sen}x}{1-\cos x}\overset{\left(\frac{0}{0}\right)}{=}\lim_{x\to 0}\frac{x\operatorname{sen}x\left(1+\cos x\right)}{\left(1-\cos x\right)\left(1+\cos x\right)}=\lim_{x\to 0}\frac{x\operatorname{sen}x\left(1+\cos x\right)}{1-\cos^2 x}=$$

$$=\lim_{x\to 0}\frac{x\cancel{\operatorname{sen}x}\left(1+\cos x\right)}{\operatorname{sen}^{\cancel{2}}x}=\lim_{x\to 0}\frac{x}{\operatorname{sen}x}\times\lim_{x\to 0}\left(1+\cos x\right)=\frac{1}{\lim\limits_{x\to 0}\dfrac{\operatorname{sen}x}{x}}\times\left(1+1\right)=$$

$$=\frac{1}{1}\times 2=2$$

3.36 $\lim\limits_{x\to -\pi}\dfrac{\cos\left(\dfrac{\pi}{2}-x\right)}{3x+3\pi}$

$$\lim_{x\to -\pi}\frac{\cos\left(\frac{\pi}{2}-x\right)}{3x+3\pi}\overset{\left(\frac{0}{0}\right)}{\underset{\substack{x\to-\pi\Rightarrow x+\pi\to 0\\ y=x+\pi\Leftrightarrow x=y-\pi\\ y\to 0}}{=}}\lim_{y\to 0}\frac{\cos\left(\frac{\pi}{2}-\left(y-\pi\right)\right)}{3\left(y-\pi\right)+3\pi}=\lim_{y\to 0}\frac{\cos\left(\frac{3}{2}\pi-y\right)}{3y}=$$

$$=\frac{1}{3}\lim_{y\to 0}\frac{\operatorname{sen}\left(-y\right)}{y}=-\frac{1}{3}\lim_{y\to 0}\frac{\operatorname{sen}y}{y}=-\frac{1}{3}\times 1=-\frac{1}{3}$$

3.37 $\lim\limits_{x\to\frac{\pi}{8}}\dfrac{\operatorname{sen}\left(\dfrac{5}{4}\pi-2x\right)}{2x-\dfrac{\pi}{4}}$

$$\lim_{x\to\frac{\pi}{8}}\frac{\operatorname{sen}\left(\frac{5}{4}\pi-2x\right)}{2x-\frac{\pi}{4}}\overset{\left(\frac{0}{0}\right)}{\underset{\substack{x\to\frac{\pi}{8}\Rightarrow x-\frac{\pi}{8}\to 0\\ y=x-\frac{\pi}{8}\Leftrightarrow x=y+\frac{\pi}{8}\\ y\to 0}}{=}}\lim_{y\to 0}\frac{\operatorname{sen}\left(\frac{5}{4}\pi-2\left(y+\frac{\pi}{8}\right)\right)}{2\left(y+\frac{\pi}{8}\right)-\frac{\pi}{4}}=$$

$$=\lim_{y\to 0}\frac{\operatorname{sen}\left(\frac{5}{4}\pi-2y-\frac{\pi}{4}\right)}{2y+\cancel{\frac{\pi}{4}}-\cancel{\frac{\pi}{4}}}=\lim_{y\to 0}\frac{\operatorname{sen}\left(\pi-2y\right)}{2y}=\lim_{y\to 0}\frac{\operatorname{sen}\left(2y\right)}{2y}\underset{y\to 0\Rightarrow 2y\to 0}{=}$$

$$=\lim_{2y\to 0}\frac{\operatorname{sen}\left(2y\right)}{2y}=1$$

3.38 $\lim\limits_{x\to\frac{\pi}{6}} \dfrac{\sqrt{3}\operatorname{sen} x - \cos x}{\cos\left(2x + \frac{\pi}{6}\right)}$

$$\lim_{x\to\frac{\pi}{6}} \frac{\sqrt{3}\operatorname{sen} x - \cos x}{\cos\left(2x+\frac{\pi}{6}\right)} \overset{\left(\frac{0}{0}\right)}{=} \lim_{x\to\frac{\pi}{6}} \frac{2\left(\frac{\sqrt{3}}{2}\operatorname{sen} x - \frac{1}{2}\cos x\right)}{\cos\left(2x+\frac{\pi}{6}\right)} =$$

$$= 2\lim_{x\to\frac{\pi}{6}} \frac{\cos\frac{\pi}{6}\times\operatorname{sen} x - \operatorname{sen}\frac{\pi}{6}\times\cos x}{\cos\left(2x+\frac{\pi}{6}\right)} = 2\lim_{x\to\frac{\pi}{6}} \frac{\operatorname{sen}\left(x-\frac{\pi}{6}\right)}{\cos\left(2x+\frac{\pi}{6}\right)} =$$

$x\to\frac{\pi}{6} \Rightarrow x-\frac{\pi}{6}\to 0$

$y = x-\frac{\pi}{6} \Leftrightarrow x = y+\frac{\pi}{6}$

$y\to 0$

$$= 2\lim_{y\to 0} \frac{\operatorname{sen}\left(y+\frac{\pi}{6}-\frac{\pi}{6}\right)}{\cos\left(2\left(y+\frac{\pi}{6}\right)+\frac{\pi}{6}\right)} = 2\lim_{y\to 0}\frac{\operatorname{sen} y}{\cos\left(2y+\frac{\pi}{2}\right)} =$$

$$= 2\lim_{y\to 0}\frac{\operatorname{sen} y}{-\operatorname{sen}(2y)} = -2\lim_{y\to 0}\frac{\operatorname{sen} y}{2\operatorname{sen} y\cos y} = -2\times\frac{1}{2\times 1} = -1$$

3.39 $\lim\limits_{x\to\frac{\pi}{4}} \dfrac{1-\tan x}{\operatorname{sen} x - \cos x}$

$$\lim_{x\to\frac{\pi}{4}} \frac{1-\tan x}{\operatorname{sen} x-\cos x} \overset{\left(\frac{0}{0}\right)}{=} \lim_{x\to\frac{\pi}{4}} \frac{1-\frac{\operatorname{sen} x}{\cos x}}{\operatorname{sen} x-\cos x} = \lim_{x\to\frac{\pi}{4}} \frac{\frac{\cos x-\operatorname{sen} x}{\cos x}}{\operatorname{sen} x-\cos x} =$$

$$= -\lim_{x\to\frac{\pi}{4}} \frac{\frac{\operatorname{sen} x-\cos x}{\cos x}}{\operatorname{sen} x-\cos x} = -\lim_{x\to\frac{\pi}{4}}\frac{1}{\cos x} = -\frac{1}{\frac{\sqrt{2}}{2}} = -\sqrt{2}$$

3.40 $\lim\limits_{x\to 0^+} \dfrac{1-\cos x}{x^3}$

$$\lim_{x\to 0^+} \frac{1-\cos x}{x^3}^{\left(\frac{0}{0}\right)} =$$

$\cos x = \cos\left(2\times\frac{x}{2}\right) = \cos^2\left(\frac{x}{2}\right) - \text{sen}^2\left(\frac{x}{2}\right) = 1 - \text{sen}^2\left(\frac{x}{2}\right) - \text{sen}^2\left(\frac{x}{2}\right) = 1 - 2\,\text{sen}^2\left(\frac{x}{2}\right)$

Assim, $\cos x = 1 - 2\,\text{sen}^2\left(\frac{x}{2}\right) \Leftrightarrow -2\,\text{sen}^2\left(\frac{x}{2}\right) = \cos x - 1 \Leftrightarrow 2\,\text{sen}^2\left(\frac{x}{2}\right) = 1 - \cos x$

$$= \lim_{x\to 0^+} \frac{2\,\text{sen}^2\left(\frac{x}{2}\right)}{x^3} = 2\left[\lim_{x\to 0^+} \frac{\text{sen}^2\left(\frac{x}{2}\right)}{4\left(\frac{x}{2}\right)^2} \times \lim_{x\to 0^+} \frac{1}{x}\right] =$$

$$= \frac{1}{2}\left[\lim_{x\to 0^+}\left(\frac{\text{sen}\left(\frac{x}{2}\right)}{\frac{x}{2}}\right)^2 \times \lim_{x\to 0^+}\frac{1}{x}\right]_{x\to 0^+ \Rightarrow \frac{x}{2}\to 0^+} = \frac{1}{2}\left[\lim_{\frac{x}{2}\to 0^+}\left(\frac{\text{sen}\left(\frac{x}{2}\right)}{\frac{x}{2}}\right)^2 \times \lim_{x\to 0^+}\frac{1}{x}\right] =$$

$$= \frac{1}{2}\left[(1)^2 \times \frac{1}{0^+}\right] = +\infty$$

3.41 $\lim\limits_{x\to \frac{\pi}{4}} \dfrac{\sqrt{2}\cos x - 1}{4x - \pi}$

$$\lim_{x\to \frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{4x-\pi}^{\left(\frac{0}{0}\right)} = \lim_{y\to 0} \frac{\sqrt{2}\cos\left(y+\frac{\pi}{4}\right) - 1}{4\left(y+\frac{\pi}{4}\right) - \pi} =$$

$x\to\frac{\pi}{4} \Rightarrow x-\frac{\pi}{4}\to 0$
$y = x - \frac{\pi}{4} \Leftrightarrow x = y + \frac{\pi}{4}$
$y\to 0$

$$= \lim_{y\to 0} \frac{\sqrt{2}\left(\cos y \cos\frac{\pi}{4} - \text{sen}\, y\, \text{sen}\frac{\sqrt{2}}{2}\right) - 1}{4y + \pi - \pi} =$$

$$= \lim_{y\to 0} \frac{\sqrt{2}\left(\cos y \times \frac{\sqrt{2}}{2} - \text{sen}\, y \times \frac{\sqrt{2}}{2}\right) - 1}{4y} = \lim_{y\to 0}\frac{\cos y - \text{sen}\, y - 1}{4y} =$$

$$= \frac{1}{4}\left(\lim_{y\to 0}\frac{\cos y - 1}{y} - \lim_{y\to 0}\frac{\text{sen}\, y}{y}\right) = \frac{1}{4}\lim_{y\to 0}\frac{(\cos y - 1)(\cos y + 1)}{y(\cos y + 1)} - \frac{1}{4} =$$

$$= \frac{1}{4}\lim_{y\to 0}\frac{\cos^2 y - 1}{y(\cos y + 1)} - \frac{1}{4} = \frac{1}{4}\lim_{y\to 0}\frac{\text{sen}^2\, y}{y(\cos y + 1)} - \frac{1}{4} =$$

$$= \frac{1}{4}\left(\lim_{y\to 0}\frac{\text{sen}\, y}{y} \times \lim_{y\to 0}\frac{\text{sen}\, y}{\cos y + 1}\right) - \frac{1}{4} = \frac{1}{4}\times\left(1\times\frac{0}{2}\right) - \frac{1}{4} = -\frac{1}{4}$$

3.42 $\lim\limits_{x\to 0}\dfrac{x+\tan\frac{x}{2}}{x}$

$$\lim_{x\to 0}\frac{x+\tan\frac{x}{2}}{x}^{\left(\frac{0}{0}\right)}=\lim_{x\to 0}\frac{x+\dfrac{\operatorname{sen}\frac{x}{2}}{\cos\frac{x}{2}}}{x}\underset{\substack{x\to 0,\ \frac{x}{2}\to 0\\ y=\frac{x}{2}\Leftrightarrow x=2y\\ y\to 0}}{=}\lim_{y\to 0}\frac{2y+\dfrac{\operatorname{sen} y}{\cos y}}{2y}=\lim_{y\to 0}\frac{2y}{2y}+\lim_{y\to 0}\frac{\operatorname{sen} y}{2y\cos y}=$$

$$=1+\frac{1}{2}\lim_{y\to 0}\frac{\operatorname{sen} y}{y}\times\lim_{y\to 0}\frac{1}{\cos y}=1+\frac{1}{2}\times 1\times\frac{1}{1}=\frac{3}{2}$$

4. Funções exponenciais, logaritmicas e trigonométricas

4.15 $$\lim_{x\to 0}\frac{e^x-\cos x}{\operatorname{sen} x}^{\left(\frac{0}{0}\right)}=\lim_{x\to 0}\frac{e^x-1+1-\cos x}{\operatorname{sen} x}=\lim_{x\to 0}\frac{e^x-1}{\operatorname{sen} x}+\lim_{x\to 0}\frac{1-\cos x}{\operatorname{sen} x}=$$

$$=\lim_{x\to 0}\frac{\dfrac{e^x-1}{x}}{\dfrac{\operatorname{sen} x}{x}}+\lim_{x\to 0}\frac{\left(1-\cos^2 x\right)\left(1+\cos x\right)}{\operatorname{sen} x\left(1+\cos x\right)}=\frac{1}{1}+\lim_{x\to 0}\frac{1-\cos^2 x}{\operatorname{sen} x\left(1+\cos x\right)}=$$

$$=1+\lim_{x\to 0}\frac{\operatorname{sen}^{\cancel{2}} x}{\cancel{\operatorname{sen} x}\left(1+\cos x\right)}=1+\lim_{x\to 0}\frac{\operatorname{sen} x}{1+\cos x}=1+\frac{0}{1+1}=1$$