1. Considere os complexos $z_1=\frac{5}{13}+\frac{12}{13}i$ e $z_2=\operatorname{sen}\frac{\pi}{7}+i\cos\frac{\pi}{7}$

Verifique que z_1 e z_2 são complexos unitários.

$$|z_1|=\sqrt{\left(\frac{5}{13}\right)^2+\left(\frac{12}{13}\right)^2}=\sqrt{\frac{25}{169}+\frac{144}{169}}=\sqrt{\frac{169}{169}}=1$$

$$|z_2|=\sqrt{\left(\operatorname{sen}\frac{\pi}{7}\right)^2+\left(\cos\frac{\pi}{7}\right)^2}=\sqrt{1}=1$$

2. Sejam os números complexos:

$$z_1=\frac{1+i}{\sqrt{2}} \quad \text{e} \quad z_2=2\cos\frac{7\pi}{12}\left(\cos\frac{7\pi}{12}+i\operatorname{sen}\frac{7\pi}{12}\right)-1$$

Mostre que:

2.1. z_1 é um complexo unitário e indique três argumentos de z_1

$$z_1=\frac{1+i}{\sqrt{2}}=\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}=\cos\frac{\pi}{4}+i\operatorname{sen}\frac{\pi}{4}$$

z_1 é unitário porque $z_1=\cos\theta+i\operatorname{sen}\theta$, com $\theta=\frac{\pi}{4}$

Os argumentos de z_1 são da forma $\frac{\pi}{4}+2k\pi$, $k\in\mathbb{Z}$

$\frac{\pi}{4}$, $\frac{\pi}{4}+2\pi=\frac{9\pi}{4}$ e $\frac{\pi}{4}+4\pi=\frac{17\pi}{4}$, são argumentos de z_1

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2.2. z_2 é um complexo unitário e indique o argumento principal de z_2

$$z_2 = 2\cos\frac{7\pi}{12}\left(\cos\frac{7\pi}{12} + i\,\mathrm{sen}\frac{7\pi}{12}\right) - 1 = 2\cos^2\frac{7\pi}{12} + \underbrace{i2\cos\frac{7\pi}{12}\mathrm{sen}\frac{7\pi}{12}}_{2\mathrm{sen}\alpha\cos\alpha=\mathrm{sen}(2\alpha)} - \left(\cos^2\frac{7\pi}{12} + \mathrm{sen}^2\frac{7\pi}{12}\right) =$$

$$= \underbrace{\cos^2\frac{7\pi}{12} - \mathrm{sen}^2\frac{7\pi}{12}}_{\cos^2\alpha - \mathrm{sen}^2\alpha = \cos(2\alpha)} + i\,\mathrm{sen}\frac{7\pi}{6} = \cos\frac{7\pi}{6} + i\,\mathrm{sen}\frac{7\pi}{6}$$

z_2 é unitário porque $z_2 = \cos\theta + i\,\mathrm{sen}\,\theta$, com $\theta = \frac{7\pi}{6}$

$\frac{7\pi}{6} \notin \left]-\pi, \pi\right[$, logo $\frac{7\pi}{6}$ não é argumento principal de z_2

$\frac{7\pi}{6} - 2\pi = -\frac{5\pi}{6} \in \left]-\pi, \pi\right[$, logo $\mathrm{Arg}(z_2) = -\frac{5\pi}{6}$

3. Considere o número complexo:

$$z_1 = \frac{1 - \sqrt{3}\,i}{2}$$

3.1. Mostre que z_1 é um complexo unitário

$$z_1 = \frac{1 - \sqrt{3}\,i}{2} = \frac{1}{2} - i\frac{\sqrt{3}}{2} = \cos\frac{\pi}{3} - i\,\mathrm{sen}\frac{\pi}{3} = \cos\left(\frac{\pi}{3}\right) + i\,\mathrm{sen}\left(-\frac{\pi}{3}\right) = \cos\left(-\frac{\pi}{3}\right) + i\,\mathrm{sen}\left(-\frac{\pi}{3}\right)$$

Como $z_1 = \cos\theta + i\,\mathrm{sen}\,\theta$, com $\theta = -\frac{\pi}{3}$, então z_1 é unitário

3.2. Determine o argumento principal de z_1

$z_1 = \cos\left(-\frac{\pi}{3}\right) + i\,\mathrm{sen}\left(-\frac{\pi}{3}\right)$ e $-\frac{\pi}{3} \in \left]-\pi, \pi\right[$, então $\mathrm{Arg}(z_1) = -\frac{\pi}{3}$

4. Escreva na forma trigonométrica os números complexos seguintes:

4.1. $z_1 = 2\sqrt{3} - 2i$

$$|z_1| = \sqrt{\left(2\sqrt{3}\right)^2 + (-2)^2} = \sqrt{12+4} = \sqrt{16} = 4$$

Seja θ um argumento de z_1

$$\begin{cases} \tan\theta = \dfrac{-2}{2\sqrt{3}} = -\dfrac{\sqrt{3}}{3} \\ \left(2\sqrt{3}, -2\right) \in 4.^{\text{o}}\ \text{Q} \end{cases} \Rightarrow \theta = -\frac{\pi}{6}$$

$$z_1 = 4e^{i\left(-\frac{\pi}{6}\right)}$$

4.2. $z_2 = -1 + \sqrt{3}\,i$

$$|z_2| = \sqrt{(-1)^2 + \left(\sqrt{3}\right)^2} = \sqrt{1+3} = \sqrt{4} = 2$$

Seja θ um argumento de z_2

$$\begin{cases} \tan\theta = \dfrac{\sqrt{3}}{-1} = -\sqrt{3} \\ \left(-1, \sqrt{3}\right) \in 2.^{\text{o}}\ \text{Q} \end{cases} \Rightarrow \theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$z_2 = 2e^{i\frac{2\pi}{3}}$$

4.3. $z_3 = -2 - 2i$

$$|z_3| = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

Seja θ um argumento de z_3

$$\begin{cases} \tan\theta = \dfrac{-2}{-2} = 1 \\ (-2, -2) \in 3.^{\text{o}}\ \text{Q} \end{cases} \Rightarrow \theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

$$z_3 = 2\sqrt{2}e^{i\frac{5\pi}{4}}$$

4.4. $z_4 = 3 + \sqrt{3}\,i$

$$|z_4| = \sqrt{(3)^2 + \left(\sqrt{3}\right)^2} = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3}$$

Seja θ um argumento de z_3

$$\begin{cases} \tan\theta = \dfrac{\sqrt{3}}{3} \\ \left(3, \sqrt{3}\right) \in 1.^{\text{o}}\ \text{Q} \end{cases} \Rightarrow \theta = \frac{\pi}{6}$$

$$z_4 = 2\sqrt{3}e^{i\frac{\pi}{6}}$$

4.5. $z_5 = \dfrac{\sqrt{3}\,i - 1}{4}$

$$z_5 = \frac{\sqrt{3}\,i - 1}{4} = -\frac{1}{4} + \frac{\sqrt{3}}{4}i$$

$$|z_5| = \sqrt{\left(-\frac{1}{4}\right)^2 + \left(\frac{\sqrt{3}}{4}\right)^2} = \sqrt{\frac{1}{16} + \frac{3}{16}} = \sqrt{\frac{4}{16}} = \frac{2}{4} = \frac{1}{2}$$

Seja θ um argumento de z_3

$$\begin{cases} \tan\theta = \dfrac{\frac{\sqrt{3}}{4}}{-\frac{1}{4}} = -\sqrt{3} \\ \left(-\dfrac{1}{4}, \dfrac{\sqrt{3}}{4}\right) \in 2.^{\text{o}}\ \text{Q} \end{cases} \Rightarrow \theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$z_5 = \frac{1}{2}e^{i\frac{2\pi}{3}}$$

4.6. $z_6 = 2 + \left(1 + i^{43}\right)^2$

$$z_6 = 2 + \left(1 + i^{43}\right)^2 = 2 + \left(1 + i^3\right)^2 = 2 + (1 - i)^2 = 2 + (1 - 2i - 1) = 2 - 2i$$

$\boxed{43 = 4 \times 10 + 3}$

$$|2 - 2i| = \sqrt{2^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$$

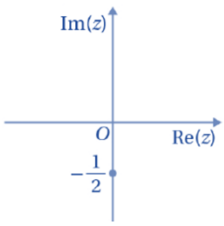
Seja θ um argumento de z_6

$$\begin{cases} \tan\theta = \dfrac{-2}{2} = -1 \\ (2, -2) \in 4.^{\text{o}}\,\text{Q} \end{cases} \Rightarrow \theta = -\frac{\pi}{4}$$

$$z_6 = 2\sqrt{2}e^{-i\frac{\pi}{4}}$$

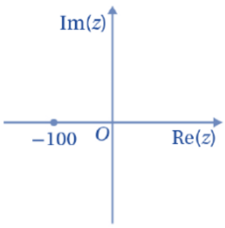
5. Represente na forma trigonométrica os números complexos seguintes.

5.1. $z = -\dfrac{1}{2}i$



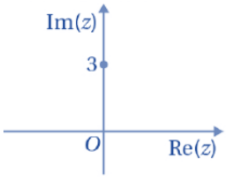
$$z = -\frac{1}{2}i = \frac{1}{2}e^{-i\frac{\pi}{2}}$$

5.2. $z = -100$



$$z = -100 = 100e^{i\pi}$$

5.3. $z=3i^5$



$z=3i^5=3i=3e^{i\pi}$

$\boxed{\begin{array}{l}\text{C.A.}\\ 5=4\times1+1\end{array}}$

5.4. $z=\dfrac{2}{i}$

$z=\dfrac{2}{i}=\dfrac{2i}{-1}=-2i=2e^{-i\frac{\pi}{2}}$

5.5. $z=|1+i|\times i$

$z=|1+i|\times i=\sqrt{1^2+1^2}\ i=\sqrt{2}\ i=\sqrt{2}\ e^{i\frac{\pi}{2}}$

6. Escreva na forma algébrica os números complexos seguintes.

6.1. $z=12e^{i\frac{\pi}{4}}$

$z=12e^{i\frac{\pi}{4}}=12\left(\cos\dfrac{\pi}{4}+i\operatorname{sen}\dfrac{\pi}{4}\right)=12\left(\dfrac{\sqrt{2}}{2}+i\dfrac{\sqrt{2}}{2}\right)=6\sqrt{2}+6\sqrt{2}\ i$

6.2. $z=e^{i\frac{17\pi}{3}}$

$z=e^{i\frac{17\pi}{3}}=e^{i\left(6\pi-\frac{\pi}{3}\right)}=e^{i\left(-\frac{\pi}{3}\right)}=\cos\left(-\dfrac{\pi}{3}\right)+i\operatorname{sen}\left(-\dfrac{\pi}{3}\right)=\cos\left(\dfrac{\pi}{3}\right)-i\operatorname{sen}\left(\dfrac{\pi}{3}\right)=\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i$

6.3. $z=\sqrt{2}e^{i\frac{\pi}{2}}$

$z=\sqrt{2}e^{i\frac{\pi}{2}}=\sqrt{2}\left(\cos\dfrac{\pi}{2}+i\operatorname{sen}\dfrac{\pi}{2}\right)=\sqrt{2}(0+i)=\sqrt{2}\ i$

6.4. $z=2e^{-i\frac{19\pi}{3}}$

$$z=2e^{-i\frac{19\pi}{3}}=2e^{-i\left(6\pi+\frac{\pi}{3}\right)}=2e^{i\left(-\frac{\pi}{3}\right)}=2\left(\cos\left(-\frac{\pi}{3}\right)+i\,\text{sen}\left(-\frac{\pi}{3}\right)\right)=2\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)=1-\sqrt{3}i$$

6.5. $z=2\sqrt{3}e^{i\frac{13\pi}{6}}$

$$z=2\sqrt{3}e^{i\frac{13\pi}{6}}=2\sqrt{3}\,e^{i\left(2\pi+\frac{\pi}{6}\right)}=2\sqrt{3}\,e^{i\frac{\pi}{6}}=2\sqrt{3}\left(\cos\frac{\pi}{6}+i\,\text{sen}\frac{\pi}{6}\right)=2\sqrt{3}\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)=3+\sqrt{3}i$$

6.6. $z=\sqrt{2}\,e^{-i\frac{21\pi}{4}}$

$$z=\sqrt{2}\,e^{-i\frac{21\pi}{4}}=\sqrt{2}e^{-i\left(\frac{24\pi}{4}-\frac{3\pi}{4}\right)}=\sqrt{2}e^{i\frac{3\pi}{4}}=\sqrt{2}\left(\cos\frac{3\pi}{4}+i\,\text{sen}\frac{3\pi}{4}\right)=\sqrt{2}\left(-\frac{\sqrt{2}}{2}+i\,\text{sen}\frac{\sqrt{2}}{2}\right)=-1+i$$

7. Mostre que $z_1=z_2$, sendo $z_1=2e^{i\frac{10\pi}{3}}$ e $z_2=2e^{-i\frac{2\pi}{3}}$

$|z_1|=|z_2|=2$

$$\frac{10\pi}{3}=-\frac{2\pi}{3}+2k\pi \Leftrightarrow \frac{10}{3}=-\frac{2}{3}+2k \Leftrightarrow 10=-2+6k \Leftrightarrow 6k=12 \Leftrightarrow k=2$$

Como $|z_1|=|z_2|$ e $\frac{10\pi}{3}=-\frac{2\pi}{3}+4\pi$, então, $z_1=z_2$

8. Verifique se são iguais os números complexos $z_1=3e^{i\frac{29\pi}{3}}$ e $z_2=3e^{i\frac{8\pi}{3}}$

$|z_1|=|z_2|=3$

$$\frac{29\pi}{6}=\frac{8\pi}{3}+2k\pi \Leftrightarrow \frac{29}{6}=\frac{8}{3}+2k \Leftrightarrow 29=16+12k \Leftrightarrow k=\frac{13}{12}\notin\mathbb{Z}$$

Logo, $z_1\neq z_2$

9. Escreva na forma trigonométrica, o simétrico e o conjugado dos números complexos seguintes.

$$z_1 = 2e^{-i\frac{\pi}{6}} \text{ e } z_2 = 3e^{i\frac{2\pi}{3}}$$

$$-z_1 = 2e^{-i\frac{\pi}{6}} = -2e^{-i\frac{\pi}{6}} = 2e^{i\left(-\frac{\pi}{6}+\pi\right)} = 2e^{i\frac{5\pi}{6}}$$

$$\overline{z_1} = \overline{2e^{-i\frac{\pi}{6}}} = 2e^{i\frac{\pi}{6}}$$

$$-z_2 = -3\,e^{i\frac{2\pi}{3}} = 3\,e^{i\left(\frac{2\pi}{3}+\pi\right)} = 3\,e^{i\frac{5\pi}{3}}$$

$$\overline{z_2} = \overline{3\,e^{i\frac{2\pi}{3}}} = 3\,e^{-i\frac{2\pi}{3}}$$

Se $z = |z|e^{i\theta}$:
- $\overline{z} = |z|e^{-i\theta}$
- $-z = |z|e^{i(\theta+\pi)}$

Se θ é um argumento de z:
- $-\theta$ é um argumento de $\overline{z}$;
- $\theta+\pi$ é um argumento de $-z$.

10. Calcule

10.1. $(1+i)^6$

Seja, $z = 1+i$

$|z| = \sqrt{1+1} = \sqrt{2}$

$$\begin{cases} \tan\theta = \frac{1}{1} = 1 \\ (1,1) \in 1.^{\circ}\mathrm{Q} \end{cases} \Rightarrow \theta = \frac{\pi}{4}, \operatorname{Arg}(z) = \frac{\pi}{4}$$

Pelo que, $z = \sqrt{2}\,e^{i\frac{\pi}{4}}$

$$(1+i)^6 = z^6 = \left(\sqrt{2}\,e^{i\frac{\pi}{4}}\right)^6 = \left(\sqrt{2}\right)^6 e^{i\frac{6\pi}{4}} = 2^{\frac{6}{2}} e^{i\frac{3\pi}{2}} = 8e^{i\frac{3\pi}{2}} = -8i$$

10.2. $\left(\sqrt{3}-i\right)^6$

Seja, $z = \sqrt{3}-i$

$|z| = \sqrt{\left(\sqrt{3}\right)^2 + (-1)^2} = \sqrt{4} = 2$

$$\begin{cases} \tan\theta = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3} \\ \left(\sqrt{3}, -1\right) \in 4.^{\circ}\mathrm{Q} \end{cases} \Rightarrow \theta = -\frac{\pi}{6}, \operatorname{Arg}(z) = -\frac{\pi}{6}$$

Pelo que, $z=2e^{i\left(-\frac{\pi}{6}\right)}$

$$\left(\sqrt{3}-i\right)^6=z^6=\left(2e^{i\left(-\frac{\pi}{6}\right)}\right)^6=2^6e^{i\left(-\frac{6\pi}{6}\right)}=64e^{i(-\pi)}=64e^{-i\pi}=-64$$

10.3. $\dfrac{\left(\sqrt{3}+3i\right)^6}{(2i)^5}$

Seja, $z=\sqrt{3}+3i$

$|z|=\sqrt{\left(\sqrt{3}\right)^2+3^2}=\sqrt{12}=2\sqrt{3}$

$$\begin{cases}\tan\theta=\dfrac{3}{\sqrt{3}}=\sqrt{3}\\ \left(\sqrt{3},3\right)\in 1.^{\circ}\text{Q}\end{cases}\Rightarrow\theta=\frac{\pi}{3},\ \operatorname{Arg}(z)=\frac{\pi}{3}$$

Pelo que, $z=2\sqrt{3}\,e^{i\frac{\pi}{3}}$

$$\left(\sqrt{3}+3i\right)^6=z^6=\left(2\sqrt{3}\,e^{i\frac{\pi}{3}}\right)^6=\left(2\sqrt{3}\right)^6e^{i\left(\frac{6\pi}{3}\right)}=1728e^{i2\pi}=1728e^{i0}=1728$$

Seja, $w=(2i)^5=2^5i^5=32i$

$$\frac{\left(\sqrt{3}+3i\right)^6}{(2i)^5}=\frac{1728}{32i}=\frac{1728i}{32i^2}=\frac{1728i}{-32}=-54i$$

10.4. $(1+i)^{10}-(1-i)^{10}$

Seja, $z=1+i$

$|z|=\sqrt{1+1}=\sqrt{2}$

$$\begin{cases}\tan\theta=\dfrac{1}{1}=1\\ (1,1)\in 1.^{\circ}\text{Q}\end{cases}\Rightarrow\theta=\frac{\pi}{4},\ \operatorname{Arg}(z)=\frac{\pi}{4}$$

Pelo que, $z=\sqrt{2}\,e^{i\frac{\pi}{4}}$

$$(1+i)^{10}=z^{10}=\left(\sqrt{2}\,e^{i\frac{\pi}{4}}\right)^{10}=\left(\sqrt{2}\right)^{10}e^{i\frac{10\pi}{4}}=2^{\frac{10}{2}}e^{i\frac{5\pi}{2}}=32e^{i\frac{\pi}{2}}=32i$$

Seja, $w = 1 - i$

$|w| = \sqrt{1 + (-1)^2} = \sqrt{2}$

$$\begin{cases} \tan\theta = \dfrac{-1}{1} = -1 \\ (1, -1) \in 4.^{\text{o}}\text{Q} \end{cases} \Rightarrow \theta = -\frac{\pi}{4}, \text{Arg}(w) = -\frac{\pi}{4}$$

Pelo que, $w = \sqrt{2}e^{-i\frac{\pi}{4}}$

$$(1-i)^{10} = w^{10} = \left(\sqrt{2}e^{-i\frac{\pi}{4}}\right)^{10} = \left(\sqrt{2}\right)^{10} e^{i\left(-\frac{10\pi}{4}\right)} = 2^{\frac{10}{2}} e^{i\left(-\frac{5\pi}{2}\right)} = 32e^{-i\frac{\pi}{2}} = -32i$$

Portanto, $(1+i)^{10} - (1-i)^{10} = 32i - (-32i) = 64i$

11. Apresente na forma algébrica os números complexos seguintes.

11.1. $z = \dfrac{(-2i)^3}{\left(\overline{1+i}\right)^6}$

$$z = \frac{(-2i)^3}{\left(\overline{1+i}\right)^6} = \frac{-8i^3}{(1-i)^6} = \frac{8i}{(1-i)^6}$$

Seja, $w = 1 - i$

$|w| = \sqrt{1 + (-1)^2} = \sqrt{2}$

$$\begin{cases} \text{Arg}(w) = \tan^{-1}\left(\dfrac{-1}{1}\right) = -\dfrac{\pi}{4} \\ (1, -1) \in 4.^{\text{o}}\text{Q} \end{cases}$$

Pelo que, $w = \sqrt{2}e^{-i\frac{\pi}{4}}$

$$(1-i)^6 = w^6 = \left(\sqrt{2}e^{-i\frac{\pi}{4}}\right)^6 = \left(\sqrt{2}\right)^6 e^{i\left(-\frac{6\pi}{4}\right)} = 2^{\frac{6}{2}} e^{i\left(-\frac{3\pi}{2}\right)} = 8e^{i\frac{\pi}{2}} = 8i$$

Portanto, $z = \dfrac{(-2i)^3}{\left(\overline{1+i}\right)^6} = \dfrac{8i}{8i} = 1$

11.2. $z=\dfrac{\left(\sqrt{3}-i\right)^6-(4i)^3}{\left(i^{43}+1\right)^4\times e^{-i\frac{\pi}{12}}}$

Seja, $z_1=\sqrt{3}-i$

$|z_1|=\sqrt{\left(\sqrt{3}\right)^2+(-1)^2}=\sqrt{4}=2$

$$\begin{cases}\tan\theta=\dfrac{-1}{\sqrt{3}}=-\dfrac{\sqrt{3}}{3}\\ \left(\sqrt{3},-1\right)\in 4.^{\text{o}}\text{Q}\end{cases}\Rightarrow \theta=-\frac{\pi}{6},\ \text{Arg}(z_1)=-\frac{\pi}{6}$$

Pelo que, $z_1=2e^{i\left(-\frac{\pi}{6}\right)}$

$$\left(\sqrt{3}-i\right)^6=z^6=\left(2e^{i\left(-\frac{\pi}{6}\right)}\right)^6=2^6e^{i\left(-\frac{6\pi}{6}\right)}=64e^{i(-\pi)}=64e^{-i\pi}=-64$$

Seja, $z_2=(4i)^3=4^3i^3=-64i$

Seja, $z_3=i^{43}+1=1+i^3=1-i$

$|z_3|=\sqrt{1+(-1)^2}=\sqrt{2}$

$$\begin{cases}\tan\theta=\dfrac{-1}{1}=-1\\ (1,-1)\in 4.^{\text{o}}\text{Q}\end{cases}\Rightarrow \theta=-\frac{\pi}{4},\ \text{Arg}(z_3)=-\frac{\pi}{4}$$

Pelo que, $z_3=\sqrt{2}e^{-i\frac{\pi}{4}}$

$$(1-i)^4={z_3}^4=\left(\sqrt{2}e^{-i\frac{\pi}{4}}\right)^4=\left(\sqrt{2}\right)^4e^{i\left(-\frac{4\pi}{4}\right)}=2^{\frac{4}{2}}e^{i(-\pi)}=4e^{-i\pi}=-4$$

Logo, $z=\dfrac{\left(\sqrt{3}-i\right)^6-(4i)^3}{\left(i^{43}+1\right)^4\times e^{-i\frac{\pi}{12}}}=\dfrac{-64-(-64i)}{-4\times e^{-i\frac{\pi}{12}}}=\dfrac{-64+64i}{-4\times e^{-i\frac{\pi}{12}}}$

Seja, $z_4=-64+64i=64(-1+i)$

$|z_4|=\sqrt{(-1)^2+1^2}=\sqrt{2}$

$$\begin{cases}\operatorname{Arg}(z_4)=\tan^{-1}(-1)=\dfrac{3\pi}{4}\\ (-1,1)\in 2.^{\text{o}}\text{Q}\end{cases}$$

Assim, $z_4=64\times\sqrt{2}\,e^{i\frac{3\pi}{4}}$

Portanto, Logo, $z=\dfrac{-64+64i}{-4\times e^{-i\frac{\pi}{12}}}=\dfrac{64\sqrt{2}\,e^{i\frac{3\pi}{4}}}{-4\times e^{-i\frac{\pi}{12}}}=-16\sqrt{2}\,e^{i\left(\frac{3\pi}{4}-\left(-\frac{\pi}{12}\right)\right)}=-16\sqrt{2}\,e^{i\left(\frac{3\pi}{4}+\frac{\pi}{12}\right)}=$

$$=-16\sqrt{2}\,e^{i\frac{10\pi}{12}}=-16\sqrt{2}\,e^{i\frac{5\pi}{6}}=-16\sqrt{2}\left(\cos\frac{5\pi}{6}+i\operatorname{sen}\frac{5\pi}{6}\right)=$$

$$=-16\sqrt{2}\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)=8\sqrt{6}-8\sqrt{2}\,i$$

12. Calcule e apresente o resultado na forma trigonométrica

12.1. $z=\left(\sqrt{3}-i\right)^2+2e^{-i\frac{\pi}{3}}$

Seja, $w=\sqrt{3}-i$

$|w|=\sqrt{\left(\sqrt{3}\right)^2+(-1)^2}=\sqrt{4}=2$

$$\begin{cases}\tan\theta=\dfrac{-1}{\sqrt{3}}=-\dfrac{\sqrt{3}}{3}\\ \left(\sqrt{3},-1\right)\in 4.^{\text{o}}\text{Q}\end{cases}\Rightarrow\theta=-\frac{\pi}{6},\ \operatorname{Arg}(w)=-\frac{\pi}{6}$$

Pelo que, $w=2e^{i\left(-\frac{\pi}{6}\right)}$

$$\left(\sqrt{3}-i\right)^2=z^6=\left(2e^{i\left(-\frac{\pi}{6}\right)}\right)^2=2^2e^{i\left(-\frac{2\pi}{6}\right)}=4e^{i\left(-\frac{\pi}{3}\right)}=4e^{-i\frac{\pi}{3}}$$

Portanto, $z=\left(\sqrt{3}-i\right)^2+2e^{-i\frac{\pi}{3}}=4e^{-i\frac{\pi}{3}}+2e^{-i\frac{\pi}{3}}=(4+2)e^{-i\frac{\pi}{3}}=6e^{-i\frac{\pi}{3}}$

12.2. $z=\overline{1+i}+2\sqrt{2}\,e^{i\frac{5\pi}{6}}\times e^{-i\frac{\pi}{12}}$

$$z=\overline{1+i}+2\sqrt{2}\,e^{i\frac{5\pi}{6}}\times e^{-i\frac{\pi}{12}}=1-i+2\sqrt{2}\,e^{i\left(\frac{5\pi}{6}-\frac{\pi}{12}\right)}=1-i+2\sqrt{2}\,e^{i\frac{9\pi}{12}}=1-i+2\sqrt{2}\,e^{i\frac{3\pi}{4}}=$$

$$=1-i+2\sqrt{2}\left(\cos\frac{3\pi}{4}+i\operatorname{sen}\frac{3\pi}{4}\right)=1-i+2\sqrt{2}\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)=1-i+-2+2i=-1+i=\sqrt{2}\,e^{i\frac{3\pi}{4}}$$

C.A.

$-1+i$

$|-1+i|=\sqrt{2}$

$\operatorname{Arg}(-1+i)=\tan^{-1}\left(\frac{1}{-1}\right)=\frac{3\pi}{4},\ (-1,1)\in 2.^{\text{o}}\text{Q}$

13. Considere os números complexos:

$$z_1=2e^{i\frac{\pi}{4}}\quad \text{e}\quad z_2=4\left(\cos\frac{\pi}{8}+i\operatorname{sen}\frac{\pi}{8}\right)$$

Determine os números complexos apresentando o resultado na forma trigonométrica.

$$z_2=4\left(\cos\frac{\pi}{8}+i\operatorname{sen}\frac{\pi}{8}\right)=4e^{i\frac{\pi}{8}}$$

13.1. $z_1\times\overline{z_2}$

$$z_1\times\overline{z_2}=2e^{i\frac{\pi}{4}}\times\overline{4e^{i\frac{\pi}{8}}}=2e^{i\frac{\pi}{4}}\times4e^{-i\frac{\pi}{8}}=8e^{i\left(\frac{\pi}{4}-\frac{\pi}{8}\right)}=8e^{i\frac{\pi}{8}}$$

13.2. $-z_1\times z_2^2$

$$-z_1\times z_2^2=-2e^{i\frac{\pi}{4}}\times\left(4e^{i\frac{\pi}{8}}\right)^2=2e^{i\left(\frac{\pi}{4}+\pi\right)}\times4^2e^{i\frac{2\pi}{8}}=2e^{i\frac{5\pi}{4}}\times16e^{i\frac{\pi}{4}}=32e^{i\left(\frac{5\pi}{4}+\frac{\pi}{4}\right)}=32e^{i\frac{3\pi}{2}}=32e^{-i\frac{\pi}{2}}$$

13.3. $\left(\frac{i\overline{z_1^2}}{z_2^2}\right)$

$$\left(\frac{i\overline{z_1^2}}{z_2^2}\right)=\frac{e^{i\frac{\pi}{2}}\times\overline{\left(\left(2e^{i\frac{\pi}{4}}\right)^2\right)}}{\left(4e^{i\frac{\pi}{8}}\right)^2}=\frac{e^{i\frac{\pi}{2}}\times\overline{\left(4e^{i\frac{\pi}{2}}\right)}}{16e^{i\frac{\pi}{4}}}=\frac{e^{i\frac{\pi}{2}}\times4e^{-i\frac{\pi}{2}}}{16e^{i\frac{\pi}{4}}}=\frac{4e^{i\left(\frac{\pi}{2}-\frac{\pi}{2}\right)}}{16e^{i\frac{\pi}{4}}}=\frac{4\times1}{16e^{i\frac{\pi}{4}}}=\frac{1}{4e^{i\frac{\pi}{4}}}=\frac{1}{4}e^{-i\frac{\pi}{4}}$$

14. Determine o módulo e o argumento principal do número complexo $z=\dfrac{\left(1-i\sqrt{3}\right)^5}{(-1+i)^4\times 2i^3}$

Seja, $w=1-\sqrt{3}\,i$

$|w|=\sqrt{1+3}=2$

$$\begin{cases}\tan\theta=\dfrac{-\sqrt{3}}{1}=-\sqrt{3}\\ \left(1,-\sqrt{3}\right)\in 4.^{\text{o}}\text{Q}\end{cases}\Rightarrow \operatorname{Arg}(w)=-\frac{\pi}{3}$$

$w=2e^{-i\frac{\pi}{3}}$

Seja, $u=-1+i$

$|u|=\sqrt{1+1}=\sqrt{2}$

$$\begin{cases}\tan\theta=\dfrac{1}{-1}=-1\\ (-1,1)\in 2.^{\text{o}}\text{Q}\end{cases}\Rightarrow \operatorname{Arg}(u)=\frac{3\pi}{4}$$

$u=\sqrt{2}\,e^{i\frac{3\pi}{4}}$

Seja, $v=2i^3=-2i=2e^{-i\frac{\pi}{2}}$

Portanto, $z=\dfrac{\left(1-i\sqrt{3}\right)^5}{(-1+i)^4\times 2i^3}=\dfrac{\left(2e^{-i\frac{\pi}{3}}\right)^5}{\left(\sqrt{2}\,e^{i\frac{3\pi}{4}}\right)^4\times 2e^{-i\frac{\pi}{2}}}=\dfrac{32e^{-i\frac{5\pi}{3}}}{4e^{i\frac{12\pi}{4}}\times 2e^{-i\frac{\pi}{2}}}=\dfrac{32e^{-i\frac{5\pi}{3}}}{4e^{i3\pi}\times 2e^{-i\frac{\pi}{2}}}=$

$=\dfrac{32e^{-i\frac{5\pi}{3}}}{8e^{i\left(3\pi-\frac{\pi}{2}\right)}}=\dfrac{32e^{-i\frac{5\pi}{3}}}{8e^{i\frac{5\pi}{2}}}=4e^{i\left(-\frac{5\pi}{3}-\frac{5\pi}{2}\right)}=4e^{-i\frac{25\pi}{6}}=4e^{-i\left(\frac{24\pi}{6}+\frac{\pi}{6}\right)}=4e^{-i\frac{\pi}{6}}$

$|z|=4$ e $\operatorname{Arg}(z)=-\dfrac{\pi}{6}$

15. Considere o número complexo $z=(1+i)\sqrt{2}\,e^{i\frac{\pi}{3}}$

Determine o menor valor de $n \in IN$ para o qual o número complexo z^n é:

Seja, $w=1+i$

$|w|=\sqrt{1+1}=\sqrt{2}$

$$\begin{cases} \tan\theta=\frac{1}{1}=1 \\ (1,1)\in 1.^{\text{o}}\text{Q} \end{cases} \Rightarrow \theta=\frac{\pi}{4}, \operatorname{Arg}(z)=\frac{\pi}{4}$$

Então, $w=\sqrt{2}\,e^{i\frac{\pi}{4}}$

$$z=(1+i)\sqrt{2}\,e^{i\frac{\pi}{3}}=\sqrt{2}\,e^{i\frac{\pi}{4}}\times\sqrt{2}\,e^{i\frac{\pi}{3}}=2e^{i\left(\frac{\pi}{4}+\frac{\pi}{3}\right)}=2e^{i\frac{7\pi}{12}}$$

$$z_n=\left(2e^{i\frac{7\pi}{12}}\right)^n=2^n e^{i\frac{7n\pi}{12}}$$

15.1. Um número real negativo.

$$\begin{aligned} z^n \in IR^- &\Leftrightarrow \frac{7n\pi}{12}=\pi+2k\pi\ ,\ k\in\mathbb{Z} \\ &\Leftrightarrow 7n\pi=12\pi+24k\pi\ ,\ k\in\mathbb{Z} \\ &\Leftrightarrow 7n=12+24k\ ,\ k\in\mathbb{Z} \\ &\Leftrightarrow n=\frac{12+24k}{7}\ ,\ k\in\mathbb{Z} \end{aligned}$$

$k=1$

$n=\frac{12+24}{7}=\frac{38}{7}\notin IN$

$k=2$

$n=\frac{12+48}{7}=\frac{60}{7}\notin IN$

$k=3$

$n=\frac{12+72}{7}=\frac{84}{7}=12\in IN$

O menor valor de n para o qual $z^n \in IR^-$ é 12 e obtém-se para $k=3$

15.2. Um número imaginário puro.

z^n é imaginário puro, então, $\frac{7n\pi}{12}=\frac{\pi}{2}+k\pi\ ,\ k\in\mathbb{Z} \Leftrightarrow 7n\pi=6\pi+12k\pi \Leftrightarrow 7n=6+12k\ ,\ k\in\mathbb{Z}$

$$\Leftrightarrow n=\frac{6+12k}{7}\ ,\ k\in\mathbb{Z}$$

$k=1$

$n=\frac{6+12}{7}=\frac{18}{7}\notin IN$

$k=2$

$n=\frac{6+24}{7}=\frac{30}{7}\notin IN$

$k=3$

$n=\frac{6+36}{7}=\frac{42}{7}=6\in IN$

O menor valor de n para o qual z^n é imaginário puro é 6 obtém-se para $k=3$