



1. Simplifica o mais possível as seguintes expressões.

a) $\sin(\pi - \alpha) - \sin(2\pi - \alpha) + \cos(-\alpha) + \cos(\pi - \alpha)$

$$\begin{aligned} & \text{sen}(\pi - \alpha) - \text{sen}(2\pi - \alpha) + \cos(-\alpha) + \cos(\pi - \alpha) = \\ & = \text{sen } \alpha + \text{sen } \alpha + \cos \alpha - \cos \alpha = 2 \text{sen } \alpha \end{aligned}$$

b) $\tan(\pi - \alpha) - \tan(2\pi - \alpha) - 3 \sin(-\alpha) - 2 \sin(2\pi + \alpha)$

$$\begin{aligned} & \text{tg}(\pi - \alpha) - \text{tg}(2\pi - \alpha) - 3 \text{sen}(-\alpha) - 2 \text{sen}(2\pi + \alpha) = \\ & = -\text{tg } \alpha + \text{tg } \alpha + 3 \text{sen } \alpha - 2 \text{sen } \alpha = \text{sen } \alpha \end{aligned}$$

c) $\sin(-\alpha) \sin(\pi + \alpha) - \cos(-\alpha) \cos(\pi - \alpha)$

$$\begin{aligned} & \text{sen}(-\alpha) \times \text{sen}(\pi + \alpha) - \cos(-\alpha) \times \cos(\pi - \alpha) = \\ & = -\text{sen } \alpha \times (-\text{sen } \alpha) - \cos \alpha \times (-\cos \alpha) = \text{sen}^2 \alpha + \cos^2 \alpha = 1 \end{aligned}$$

d) $5 \sin\left(\frac{\pi}{2} - \alpha\right) + \cos\left(\frac{\pi}{2} + \alpha\right) - \sin(\pi + \alpha) + 3 \cos(\pi + \alpha)$

$$\begin{aligned} & 5 \text{sen}\left(\frac{\pi}{2} - \alpha\right) + \cos\left(\frac{\pi}{2} + \alpha\right) - \text{sen}(\pi + \alpha) + 3 \cos(\pi + \alpha) = \\ & = 5 \cos \alpha - \text{sen } \alpha + \text{sen } \alpha - 3 \cos \alpha = 2 \cos \alpha \end{aligned}$$

e) $\sin\left(\frac{3\pi}{2} - \alpha\right) + \cos(2\pi - \alpha) - \tan(-\alpha) \cos(2\pi - \alpha) + \cos\left(\frac{3\pi}{2} + \alpha\right)$

$$\begin{aligned} & \text{sen}\left(\frac{3\pi}{2} - \alpha\right) + \cos(2\pi - \alpha) - \text{tg}(-\alpha) \times \cos(2\pi - \alpha) + \cos\left(\frac{3\pi}{2} + \alpha\right) = \\ & = -\cos \alpha + \cos \alpha + \text{tg } \alpha \times \cos \alpha + \text{sen } \alpha = \frac{\text{sen } \alpha}{\cos \alpha} \times \cos \alpha + \text{sen } \alpha = 2 \text{sen } \alpha \end{aligned}$$

f) $\sin(\alpha - \pi) - \tan(\alpha + \pi) - \tan(2\pi - \alpha) - 5 \cos\left(\alpha - \frac{3\pi}{2}\right)$

$$\begin{aligned} & \text{sen}(\alpha - \pi) - \text{tg}(\alpha + \pi) - \text{tg}(2\pi - \alpha) - 5 \cos\left(\alpha - \frac{3\pi}{2}\right) = \\ & = -\text{sen } \alpha - \text{tg } \alpha + \text{tg } \alpha + 5 \text{sen } \alpha = 4 \text{sen } \alpha \end{aligned}$$

g) $\sin(\alpha - 5\pi) \tan(-\alpha - 4\pi) - 3 \sin\left(\alpha - \frac{\pi}{2}\right) + 2 \cos(7\pi - \alpha)$

$$\begin{aligned} & \text{sen}(\alpha - 5\pi) \times \text{tg}(-\alpha - 4\pi) - 3 \text{sen}\left(\alpha - \frac{\pi}{2}\right) + 2 \cos(7\pi - \alpha) = \\ & = -\text{sen } \alpha \times (-\text{tg } \alpha) + 3 \cos \alpha - 2 \cos \alpha = \text{sen } \alpha \times \frac{\text{sen } \alpha}{\cos \alpha} + \cos \alpha = \\ & = \frac{\text{sen}^2 \alpha}{\cos \alpha} + \cos \alpha = \frac{\text{sen}^2 \alpha + \cos^2 \alpha}{\cos \alpha} = \frac{1}{\cos \alpha} \end{aligned}$$

h) $\cos\left(\frac{5\pi}{2} + \alpha\right) - \sin\left(\alpha - \frac{7\pi}{2}\right) - \cos(7\pi - \alpha) + 2 \sin(\alpha + 4\pi)$

$$\begin{aligned} & \cos\left(\frac{5\pi}{2} + \alpha\right) - \text{sen}\left(\alpha - \frac{7\pi}{2}\right) - \cos(7\pi - \alpha) + 2 \text{sen}(\alpha + 4\pi) = \\ & = -\text{sen } \alpha - \cos \alpha + \cos \alpha + 2 \text{sen } \alpha = \text{sen } \alpha \end{aligned}$$

2. Qual é o valor lógico da proposição:

Se $\sin(\pi - x) > 0$ e $\tan\left(-\frac{\pi}{2} - x\right) < 0$, então $\cos(-\pi - x) < 0$

$$\sin(\pi - x) > 0 \Leftrightarrow \sin x > 0$$

$\tan\left(-\frac{\pi}{2} - x\right) < 0 \Leftrightarrow \tan x < 0$ sabemos que a tangente é negativa nos 2.º e 4.º quadrantes.

Como $\sin x > 0$, então a tangente está no 2.º quadrante.

$$\text{Temos que } \cos(-\pi - x) = \cos(-(\pi + x)) = \cos(\pi + x) = -\cos x$$

No 2.º quadrante, o cosseno é negativo, então:

$$-\cos(x) < 0 \Leftrightarrow \cos(-\pi - x) < 0$$

Portanto a proposição é verdadeira.

3. Prova que, para qualquer x para o qual as expressões têm significado, se tem:

a) $1 - \frac{\cos(2\pi+x)}{\tan(2\pi+x)\sin x + \cos(2\pi-x)} = \sin^2 x$

$$\begin{aligned} 1 - \frac{\cos(2\pi+x)}{\tan(2\pi+x)\sin x + \cos(2\pi-x)} &= 1 - \frac{\cos x}{\text{tg } x \times \sin x + \cos x} = \\ &= 1 - \frac{\cos x}{\frac{\sin x}{\cos x} \times \sin x + \cos x} = 1 - \frac{\cos x}{\frac{\sin^2 x + \cos^2 x}{\cos x}} = 1 - \frac{\cos x}{\frac{1}{\cos x}} = 1 - \cos^2 x = \sin^2 x \end{aligned}$$

b) $2 \sin\left(\frac{\pi}{2} - x\right) \sin\left(\frac{\pi}{2} + x\right) - \cos(2\pi) = [1 - \sqrt{2} \sin(2\pi + x)][1 - \sqrt{2} \sin(\pi + x)]$

$$\begin{aligned} 2 \sin\left(\frac{\pi}{2} - x\right) \sin\left(\frac{\pi}{2} + x\right) - \cos(2\pi) &= 2 \cos x \cos x - \cos(2\pi) = 2 \cos^2 x - 1 = \\ &= 2(1 - \sin^2 x) - 1 = 2 - 2 \sin^2 x - 1 = 1 - 2 \sin^2 x = 1 - \left(\sqrt{2} \sin x\right)^2 = \\ &= (1 - \sqrt{2} \sin x)(1 + \sqrt{2} \sin x) = (1 - \sqrt{2} \sin(2\pi + x))(1 - \sqrt{2} \sin(\pi + x)) \end{aligned}$$

4. Determina o valor das seguintes expressões.

a) $\sin\left(-\frac{\pi}{6}\right) + \sqrt{6} \sin\left(-\frac{\pi}{4}\right) \sin\left(-\frac{\pi}{3}\right)$

$$\begin{aligned} \sin\left(-\frac{\pi}{6}\right) + \sqrt{6} \sin\left(-\frac{\pi}{4}\right) \sin\left(-\frac{\pi}{3}\right) &= \\ &= -\sin\left(\frac{\pi}{6}\right) + \left(-\sqrt{6} \sin\left(\frac{\pi}{4}\right)\right) \left(-\sin\left(\frac{\pi}{3}\right)\right) = \\ &= -\sin\left(\frac{\pi}{6}\right) + \sqrt{6} \sin\left(\frac{\pi}{4}\right) \sin\left(\frac{\pi}{3}\right) = \\ &= -\frac{1}{2} + \sqrt{6} \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} = -\frac{1}{2} + \frac{6}{4} = -\frac{1}{2} + \frac{3}{2} = 1 \end{aligned}$$

b) $\sin\left(-\frac{3\pi}{4}\right) - \cos\left(-\frac{\pi}{4}\right) + \sqrt{5 - \sqrt{3}} \tan\left(-\frac{\pi}{3}\right)$

$$\begin{aligned} & \sin\left(-\frac{3\pi}{4}\right) - \cos\left(-\frac{\pi}{4}\right) + \sqrt{5 - \sqrt{3}} \operatorname{tg}\left(-\frac{\pi}{3}\right) = \\ & = -\operatorname{sen}\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{4}\right) + \sqrt{5 + \sqrt{3}} \operatorname{tg}\left(\frac{\pi}{3}\right) = \\ & = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} + \sqrt{5 + \sqrt{3}} \times \sqrt{3} = -\sqrt{2} + \sqrt{8} = -\sqrt{2} + 2\sqrt{2} = \sqrt{2} \end{aligned}$$

c) $\frac{\sin\frac{5\pi}{6} + \cos\left(-\frac{3\pi}{4}\right)}{2 + \cos(-\pi) + \cos\left(-\frac{2\pi}{3}\right) - \sin\left(-\frac{3\pi}{4}\right)}$

$$\begin{aligned} & \frac{\operatorname{sen} \frac{5\pi}{6} + \cos\left(-\frac{3\pi}{4}\right)}{2 + \cos(-\pi) + \cos\left(-\frac{2\pi}{3}\right) - \operatorname{sen}\left(-\frac{3\pi}{4}\right)} = \frac{\operatorname{sen} \frac{\pi}{6} - \cos \frac{\pi}{4}}{2 - 1 - \cos \frac{\pi}{3} + \operatorname{sen} \frac{\pi}{4}} = \\ & = \frac{\frac{1}{2} - \frac{\sqrt{2}}{2}}{1 - \frac{1}{2} + \frac{\sqrt{2}}{2}} = \frac{\frac{1 - \sqrt{2}}{2}}{\frac{1 + \sqrt{2}}{2}} = \frac{1 - \sqrt{2}}{1 + \sqrt{2}} = \frac{(1 - \sqrt{2})(1 - \sqrt{2})}{(1 + \sqrt{2})(1 - \sqrt{2})} = \frac{1 - 2\sqrt{2} + 2}{1 - 2} = \\ & = \frac{3 - 2\sqrt{2}}{-1} = 2\sqrt{2} - 3 \end{aligned}$$

d) $\frac{\sin\left(-\frac{17\pi}{3}\right)}{\cos\frac{17\pi}{4}} + \sqrt{6} \sin\frac{37\pi}{6}$

$$\begin{aligned} & \frac{\operatorname{sen}\left(-\frac{17\pi}{3}\right)}{\cos\frac{17\pi}{4}} + \sqrt{6} \operatorname{sen} \frac{37\pi}{6} = \frac{\operatorname{sen}\left(-\frac{17\pi}{3} + 6\pi\right)}{\cos\left(\frac{17\pi}{4} - 4\pi\right)} + \sqrt{6} \operatorname{sen}\left(\frac{37\pi}{6} - 6\pi\right) = \\ & = \frac{\operatorname{sen} \frac{\pi}{3}}{\cos \frac{\pi}{4}} + \sqrt{6} \operatorname{sen} \frac{\pi}{6} = \frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{2}}{2}} + \sqrt{6} \times \frac{1}{2} = \frac{\sqrt{3}}{\sqrt{2}} + \frac{\sqrt{6}}{2} = \frac{\sqrt{6}}{2} + \frac{\sqrt{6}}{2} = \sqrt{6} \end{aligned}$$

5. Seja $x \in \left[\frac{3\pi}{2}, 2\pi\right]$, tal que $\cos\left(\frac{\pi}{2} - x\right) \tan(\pi + x) - \sin\left(\frac{3\pi}{2} - x\right) = 2$.

a) Determina o valor de x .

$$\begin{aligned} \cos\left(\frac{\pi}{2} - x\right) \times \operatorname{tg}(\pi + x) - \operatorname{sen}\left(\frac{3\pi}{2} - x\right) &= 2 \wedge x \in \left[\frac{3\pi}{2}, 2\pi\right] \Leftrightarrow \\ \Leftrightarrow \operatorname{sen} x \times \operatorname{tg} x + \cos x &= 2 \wedge x \in \left[\frac{3\pi}{2}, 2\pi\right] \Leftrightarrow \\ \Leftrightarrow \operatorname{sen} x \times \frac{\operatorname{sen} x}{\cos x} + \cos x &= 2 \wedge x \in \left[\frac{3\pi}{2}, 2\pi\right] \Leftrightarrow \\ \Leftrightarrow \frac{\operatorname{sen}^2 x + \cos^2 x}{\cos x} &= 2 \wedge x \in \left[\frac{3\pi}{2}, 2\pi\right] \Leftrightarrow \frac{1}{\cos x} = 2 \wedge x \in \left[\frac{3\pi}{2}, 2\pi\right] \Leftrightarrow \\ \Leftrightarrow 1 = 2 \cos x \wedge \cos x &\neq 0 \wedge x \in \left[\frac{3\pi}{2}, 2\pi\right] \Leftrightarrow \\ \Leftrightarrow \cos x = \frac{1}{2} \wedge x \in \left[\frac{3\pi}{2}, 2\pi\right] &\Leftrightarrow x = \frac{5\pi}{3} \end{aligned}$$

b) Determina o valor de $\tan(\pi - x) + 2 \cos(\pi + x) - 2 \sin(2\pi - x)$.

$$\begin{aligned} \operatorname{tg}\left(\pi - \frac{5\pi}{3}\right) + 2 \cos\left(\pi + \frac{5\pi}{3}\right) - 2 \operatorname{sen}\left(2\pi - \frac{5\pi}{3}\right) &= \\ = \operatorname{tg}\left(-\frac{2\pi}{3}\right) + 2 \cos \frac{8\pi}{3} - 2 \operatorname{sen} \frac{\pi}{3} &= \operatorname{tg} \frac{\pi}{3} + 2 \cos \frac{2\pi}{3} - 2 \operatorname{sen} \frac{\pi}{3} = \\ = \sqrt{3} - 2 \cos \frac{\pi}{3} - 2 \times \frac{\sqrt{3}}{2} &= \sqrt{3} - 2 \times \frac{1}{2} - \sqrt{3} = -1 \end{aligned}$$