1. Resolve em $\mathbb{R}$ as seguintes equações:

a) $3 + 4\sin x = 5$

$$3 + 4\operatorname{sen} x = 5 \Leftrightarrow 4\operatorname{sen} x = 2 \Leftrightarrow \operatorname{sen} x = \frac{1}{2} \Leftrightarrow$$
$$\Leftrightarrow x = \frac{\pi}{6} + 2k\pi \vee x = \frac{5\pi}{6} + 2k\pi,\ k \in \mathbb{Z}$$

b) $2 + \sqrt{8}\sin x = 0$

$$2 + \sqrt{8}\operatorname{sen} x = 0 \Leftrightarrow \operatorname{sen} x = \frac{-2}{\sqrt{8}} \Leftrightarrow \operatorname{sen} x = -\frac{2}{2\sqrt{2}} \Leftrightarrow \operatorname{sen} x = -\frac{1}{\sqrt{2}} \Leftrightarrow$$
$$\Leftrightarrow \operatorname{sen} x = -\frac{\sqrt{2}}{2} \Leftrightarrow x = -\frac{\pi}{4} + 2k\pi \vee x = \frac{5\pi}{4} + 2k\pi,\ k \in \mathbb{Z}$$

c) $\sqrt{3} + 2\sin 4x = \sqrt{12}$

$$\sqrt{3} + 2\operatorname{sen}(4x) = \sqrt{12} \Leftrightarrow 2\operatorname{sen}(4x) = 2\sqrt{3} - \sqrt{3} \Leftrightarrow \operatorname{sen}(4x) = \frac{\sqrt{3}}{2} \Leftrightarrow$$
$$\Leftrightarrow 4x = \frac{\pi}{3} + 2k\pi \vee 4x = \frac{2\pi}{3} + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$$
$$\Leftrightarrow x = \frac{\pi}{12} + \frac{k\pi}{2} \vee x = \frac{\pi}{6} + \frac{k\pi}{2},\ k \in \mathbb{Z}$$

d) $1 - \sin(-x) = 2$

$$1 - \operatorname{sen}(-x) = 2 \Leftrightarrow -\operatorname{sen}(-x) = 2 - 1 \Leftrightarrow \operatorname{sen}(-x) = -1 \Leftrightarrow$$
$$\Leftrightarrow -x = -\frac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow x = \frac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z}$$

e) $\frac{1-5\sin 3x}{3} = 2$

$$\frac{1-5\operatorname{sen}(3x)}{3} = 2 \Leftrightarrow 1 - 5\operatorname{sen}(3x) = 6 \Leftrightarrow -5\operatorname{sen}(3x) = 5 \Leftrightarrow \operatorname{sen}(3x) = -1 \Leftrightarrow$$
$$\Leftrightarrow 3x = \frac{3\pi}{2} + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow x = \frac{\pi}{2} + \frac{2k\pi}{3},\ k \in \mathbb{Z}$$

f) $1 + \sin 4x = \frac{3+\sin(-4x)}{3}$

$$1 + \operatorname{sen}(4x) = \frac{3 + \operatorname{sen}(-4x)}{3} \Leftrightarrow 3 + 3\operatorname{sen}(4x) = 3 + \operatorname{sen}(-4x) \Leftrightarrow$$
$$\Leftrightarrow 3\operatorname{sen}(4x) = \operatorname{sen}(-4x) \Leftrightarrow 3\operatorname{sen}(4x) = -\operatorname{sen}(4x) \Leftrightarrow 4\operatorname{sen}(4x) = 0 \Leftrightarrow$$
$$\Leftrightarrow \operatorname{sen}(4x) = 0 \Leftrightarrow 4x = k\pi,\ k \in \mathbb{Z} \Leftrightarrow x = \frac{k\pi}{4},\ k \in \mathbb{Z}$$

g) $\left(1-2\sin\frac{x}{2}\right)\left(1+\sqrt{2}\sin x\right)=0$

$$\left(1-2\,\text{sen}\,\frac{x}{2}\right)\left(1+\sqrt{2}\,\text{sen}\,x\right)=0 \Leftrightarrow 1-2\,\text{sen}\,\frac{x}{2}=0\ \vee\ 1+\sqrt{2}\,\text{sen}\,x=0 \Leftrightarrow$$

$$\Leftrightarrow \text{sen}\,\frac{x}{2}=\frac{1}{2}\ \vee\ \text{sen}\,x=-\frac{1}{\sqrt{2}} \Leftrightarrow \text{sen}\,\frac{x}{2}=\frac{1}{2}\ \vee\ \text{sen}\,x=-\frac{\sqrt{2}}{2} \Leftrightarrow$$

$$\Leftrightarrow \frac{x}{2}=\frac{\pi}{6}+2k\pi \vee \frac{x}{2}=\frac{5\pi}{6}+2k\pi \vee x=-\frac{\pi}{4}+2k\pi \vee x=\frac{5\pi}{4}+2k\pi,\ k\in\mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x=\frac{\pi}{3}+4k\pi\ \vee\ x=\frac{5\pi}{3}+4k\pi\ \vee\ x=-\frac{\pi}{4}+2k\pi\ \vee\ x=\frac{5\pi}{4}+2k\pi,\ k\in\mathbb{Z}$$

h) $2\sin^2 x+\sqrt{3}\sin x=0$

$$2\,\text{sen}^2\,x+\sqrt{3}\,\text{sen}\,x=0 \Leftrightarrow \text{sen}\,x\left(2\,\text{sen}\,x+\sqrt{3}\right)=0 \Leftrightarrow \text{sen}\,x=0\ \vee\ 2\,\text{sen}\,x+\sqrt{3}=0 \Leftrightarrow$$

$$\Leftrightarrow \text{sen}\,x=0 \vee \text{sen}\,x=-\frac{\sqrt{3}}{2} \Leftrightarrow x=k\pi \vee x=-\frac{\pi}{3}+2k\pi \vee x=\frac{4\pi}{3}+2k\pi,\ k\in$$

i) $5+4\sin^2\left(\frac{x}{4}\right)=7$

$$5+4\,\text{sen}^2\left(\frac{x}{4}\right)=7 \Leftrightarrow 4\,\text{sen}^2\left(\frac{x}{4}\right)=2 \Leftrightarrow \text{sen}^2\left(\frac{x}{4}\right)=\frac{1}{2} \Leftrightarrow \text{sen}\left(\frac{x}{4}\right)=\pm\sqrt{\frac{1}{2}} \Leftrightarrow$$

$$\Leftrightarrow \text{sen}\left(\frac{x}{4}\right)=\pm\frac{\sqrt{2}}{2} \Leftrightarrow \frac{x}{4}=\frac{\pi}{4}+k\pi \vee \frac{x}{4}=-\frac{\pi}{4}+k\pi,\ k\in\mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x=\pi+4k\pi \vee x=-\pi+4k\pi,\ k\in\mathbb{Z} \Leftrightarrow x=\pm\pi+4k\pi,\ k\in\mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x=\pi+2k\pi,\ k\in\mathbb{Z}$$

j) $2\sin^2 6x+3\sin 6x+1=0$

$$2\,\text{sen}^2\,(6x)+3\,\text{sen}\,(6x)+1=0 \Leftrightarrow \text{sen}\,(6x)=\frac{-3\pm\sqrt{3^2-4\times 2\times 1}}{2\times 2} \Leftrightarrow$$

$$\Leftrightarrow \text{sen}\,(6x)=\frac{-3\pm 1}{4} \Leftrightarrow \text{sen}\,(6x)=\frac{-3+1}{4} \vee \text{sen}\,(6x)=\frac{-3-1}{4} \Leftrightarrow$$

$$\Leftrightarrow \text{sen}\,(6x)=-\frac{1}{2} \vee \text{sen}\,(6x)=-1 \Leftrightarrow$$

$$\Leftrightarrow 6x=-\frac{\pi}{6}+2k\pi\ \vee\ 6x=\frac{7\pi}{6}+2k\pi\ \vee\ 6x=-\frac{\pi}{2}+2k\pi,\ k\in\mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x=-\frac{\pi}{36}+\frac{k\pi}{3}\ \vee\ x=\frac{7\pi}{36}+\frac{k\pi}{3}\ \vee\ x=-\frac{\pi}{12}+\frac{k\pi}{3},\ k\in\mathbb{Z}$$

k) $\sin(\pi-4x)+\sin(\pi+4x)+\cos\left(\frac{3\pi}{2}-4x\right)+\sin(2\pi-4x)=\sqrt{3}$

$$\text{sen}\,(\pi-4x)+\text{sen}\,(\pi+4x)+\cos\left(\frac{3\pi}{2}-4x\right)+\text{sen}\,(2\pi-4x)=\sqrt{3} \Leftrightarrow$$

$$\Leftrightarrow \text{sen}\,(4x)-\text{sen}\,(4x)-\text{sen}\,(4x)-\text{sen}\,(4x)=\sqrt{3} \Leftrightarrow -2\,\text{sen}\,(4x)=\sqrt{3} \Leftrightarrow$$

$$\Leftrightarrow \text{sen}\,(4x)=-\frac{\sqrt{3}}{2} \Leftrightarrow 4x=-\frac{\pi}{3}+2k\pi \vee 4x=\frac{4\pi}{3}+2k\pi,\ k\in\mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x=-\frac{\pi}{12}+\frac{k\pi}{2} \vee x=\frac{\pi}{3}+\frac{k\pi}{2},\ k\in\mathbb{Z}$$

l) $-\sqrt{3}+6\cos x=\sqrt{12}$

$$-\sqrt{3}+6\cos x=\sqrt{12}\Leftrightarrow 6\cos x=2\sqrt{3}+\sqrt{3}\Leftrightarrow \cos x=\frac{3\sqrt{3}}{6}\Leftrightarrow \cos x=\frac{\sqrt{3}}{2}\Leftrightarrow$$
$$\Leftrightarrow x=\frac{\pi}{6}+2k\pi \vee x=-\frac{\pi}{6}+2k\pi\,,\ k\in\mathbb{Z}$$

m) $5+4\cos x=3$

$$5+4\cos x=3\Leftrightarrow 4\cos x=-2\Leftrightarrow \cos x=-\frac{1}{2}\Leftrightarrow$$
$$\Leftrightarrow x=\frac{2\pi}{3}+2k\pi \vee x=\frac{4\pi}{3}+2k\pi\,,\ k\in\mathbb{Z}$$

n) $\sqrt{8}+5\cos\frac{x}{2}=3\left(\sqrt{2}+\cos\frac{x}{2}\right)$

$$\sqrt{8}+5\cos\frac{x}{2}=3\left(\sqrt{2}+\cos\frac{x}{2}\right)\Leftrightarrow 2\sqrt{2}+5\cos\frac{x}{2}=3\sqrt{2}+3\cos\frac{x}{2}\Leftrightarrow$$
$$\Leftrightarrow 2\cos\frac{x}{2}=\sqrt{2}\Leftrightarrow \cos\frac{x}{2}=\frac{\sqrt{2}}{2}\Leftrightarrow \frac{x}{2}=\frac{\pi}{4}+2k\pi \vee \frac{x}{2}=-\frac{\pi}{4}+2k\pi\,,\ k\in\mathbb{Z}\Leftrightarrow$$
$$\Leftrightarrow x=\frac{\pi}{2}+4k\pi \vee x=-\frac{\pi}{2}+4k\pi\,,\ k\in\mathbb{Z}$$

o) $\cos^2 x-\cos(\pi+x)=0$

$$\cos^2 x-\cos(\pi+x)=0\Leftrightarrow \cos^2 x+\cos x=0\Leftrightarrow \cos x\,(\cos x+1)=0\Leftrightarrow$$
$$\Leftrightarrow \cos x=0 \vee \cos x+1=0\Leftrightarrow \cos x=0 \vee \cos x=-1\Leftrightarrow$$
$$\Leftrightarrow x=\frac{\pi}{2}+k\pi \vee x=\pi+2k\pi\,,\ k\in\mathbb{Z}$$

p) $\frac{\sin^2 x}{1-\cos x}=2$

$$\frac{\operatorname{sen}^2 x}{1-\cos x}=2\Leftrightarrow \operatorname{sen}^2 x=2-2\cos x \wedge 1-\cos x\neq 0\Leftrightarrow$$
$$\Leftrightarrow 1-\cos^2 x=2-2\cos x \wedge 1-\cos x\neq 0\Leftrightarrow$$
$$\Leftrightarrow \cos^2 x-2\cos x+1=0 \wedge 1-\cos x\neq 0\Leftrightarrow (1-\cos x)^2=0 \wedge 1-\cos x\neq 0\Leftrightarrow$$
$$\Leftrightarrow 1-\cos x=0 \wedge 1-\cos x\neq 0 \ \rightarrow \text{Equação impossível}$$

q) $2\cos^3 x-\cos x=0$

$$2\cos^3 x-\cos x=0\Leftrightarrow \cos x\,(2\cos^2 x-1)=0\Leftrightarrow \cos x=0 \vee 2\cos^2 x-1=0\Leftrightarrow$$
$$\Leftrightarrow \cos x=0 \vee \cos^2 x=\frac{1}{2}\Leftrightarrow \cos x=0 \vee \cos x=\pm\sqrt{\frac{1}{2}}\Leftrightarrow$$
$$\Leftrightarrow \cos x=0 \vee \cos x=\pm\frac{\sqrt{2}}{2}\Leftrightarrow$$
$$\Leftrightarrow x=\frac{\pi}{2}+k\pi \vee x=\pm\frac{\pi}{4}+2k\pi \vee x=\pm\frac{3\pi}{4}+2k\pi\,,\ k\in\mathbb{Z}\Leftrightarrow$$
$$\Leftrightarrow x=\frac{\pi}{2}+k\pi \vee x=\frac{\pi}{4}+\frac{k\pi}{2}\,,\ k\in\mathbb{Z}$$

r) $\cos\left(5x - \frac{2\pi}{3}\right) = \cos\left(3x + \frac{4\pi}{3}\right)$

$$\cos\left(5x - \frac{2\pi}{3}\right) = \cos\left(3x + \frac{4\pi}{3}\right) \Leftrightarrow$$

$$\Leftrightarrow 5x - \frac{2\pi}{3} = 3x + \frac{4\pi}{3} + 2k\pi \ \vee \ 5x - \frac{2\pi}{3} = -3x - \frac{4\pi}{3} + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow 2x = 2\pi + 2k\pi \ \vee \ 8x = -\frac{2\pi}{3} + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x = \pi + k\pi \ \vee \ x = -\frac{\pi}{12} + \frac{k\pi}{4},\ k \in \mathbb{Z}$$

s) $\sin\left(\frac{\pi}{2} - 4x\right) - \cos(\pi - 4x) = 1 - 4\sin\left(\frac{3\pi}{2} + 4x\right)$

$$\operatorname{sen}\left(\frac{\pi}{2} - 4x\right) - \cos(\pi - 4x) = 1 - 4\operatorname{sen}\left(\frac{3\pi}{2} + 4x\right) \Leftrightarrow$$

$$\Leftrightarrow \cos(4x) + \cos(4x) = 1 + 4\cos(4x) \Leftrightarrow 2\cos(4x) = -1 \Leftrightarrow \cos(4$$

$$\Leftrightarrow 4x = \frac{2\pi}{3} + 2k\pi \ \vee \ 4x = \frac{4\pi}{3} + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{6} + \frac{k\pi}{2} \ \vee \ x = \frac{\pi}{3} + \frac{k\pi}{2},\ k \in \mathbb{Z}$$

t) $1 + 5\tan x = 6$

$$1 + 5\operatorname{tg} x = 6 \Leftrightarrow \operatorname{tg} x = 1 \Leftrightarrow x = \frac{\pi}{4} + k\pi,\ k \in \mathbb{Z}$$

u) $4 + \sqrt{3}\tan\frac{x}{3} = 7$

$$4 + \sqrt{3}\operatorname{tg}\frac{x}{3} = 7 \Leftrightarrow \operatorname{tg}\frac{x}{3} = \frac{3}{\sqrt{3}} \Leftrightarrow \operatorname{tg}\frac{x}{3} = \frac{3\sqrt{3}}{3} \Leftrightarrow \operatorname{tg}\frac{x}{3} = \sqrt{3} \Leftrightarrow$$
$$\Leftrightarrow \frac{x}{3} = \frac{\pi}{3} + k\pi,\ k \in \mathbb{Z} \Leftrightarrow x = \pi + 3k\pi,\ k \in \mathbb{Z}$$

v) $3\tan\left(5x + \frac{\pi}{4}\right) + \sqrt{12} = \sqrt{3}$

$$3\operatorname{tg}\left(5x + \frac{\pi}{4}\right) + \sqrt{12} = \sqrt{3} \Leftrightarrow 3\operatorname{tg}\left(5x + \frac{\pi}{4}\right) = \sqrt{3} - 2\sqrt{3} \Leftrightarrow$$
$$\Leftrightarrow \operatorname{tg}\left(5x + \frac{\pi}{4}\right) = -\frac{\sqrt{3}}{3} \Leftrightarrow 5x + \frac{\pi}{4} = -\frac{\pi}{6} + k\pi,\ k \in \mathbb{Z} \Leftrightarrow$$
$$\Leftrightarrow 5x = -\frac{5\pi}{12} + k\pi,\ k \in \mathbb{Z} \Leftrightarrow x = -\frac{\pi}{12} + \frac{k\pi}{5},\ k \in \mathbb{Z}$$

w) $\tan^2\frac{x}{2} = \tan\frac{x}{2}$

$$\operatorname{tg}^2\frac{x}{2} = \operatorname{tg}\frac{x}{2} \Leftrightarrow \operatorname{tg}^2\frac{x}{2} - \operatorname{tg}\frac{x}{2} = 0 \Leftrightarrow \operatorname{tg}\frac{x}{2}\left(\operatorname{tg}\frac{x}{2} - 1\right) = 0 \Leftrightarrow$$

$$\Leftrightarrow \operatorname{tg}\frac{x}{2} = 0 \ \vee \ \operatorname{tg}\frac{x}{2} - 1 = 0 \Leftrightarrow \operatorname{tg}\frac{x}{2} = 0 \ \vee \ \operatorname{tg}\frac{x}{2} = 1 \Leftrightarrow$$

$$\Leftrightarrow \frac{x}{2} = k\pi \ \vee \ \frac{x}{2} = \frac{\pi}{4} + k\pi,\ k \in \mathbb{Z} \Leftrightarrow x = 2k\pi \ \vee \ x = \frac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z}$$

x) $\tan^2 2x - 3 = 0$

$$\text{tg}^2\,(2x) - 3 = 0 \Leftrightarrow \text{tg}^2\,(2x) = 3 \Leftrightarrow \text{tg}\,(2x) = \pm\sqrt{3} \Leftrightarrow 2x = \pm\frac{\pi}{3} + k\pi,\ k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x = \pm\frac{\pi}{6} + \frac{k\pi}{2},\ k \in \mathbb{Z}$$

y) $3\tan^2 x = 3 + 2\sqrt{3}\tan x$

$$3\,\text{tg}^2\,x = 3 + 2\sqrt{3}\ \text{tg}\,x \Leftrightarrow 3\,\text{tg}^2\,x - 2\sqrt{3}\ \text{tg}\,x - 3 = 0 \Leftrightarrow \text{tg}\,x = \frac{2\sqrt{3} \pm \sqrt{12 - 4 \times 3 \times (-3)}}{2 \times 3} \Leftrightarrow$$

$$\Leftrightarrow \text{tg}\,x = \frac{2\sqrt{3} \pm \sqrt{48}}{6} \Leftrightarrow \text{tg}\,x = \frac{2\sqrt{3} \pm 4\sqrt{3}}{6} \Leftrightarrow \text{tg}\,x = \sqrt{3}\ \vee\ \text{tg}\,x = -\frac{\sqrt{3}}{3} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{3} + k\pi\ \vee\ x = -\frac{\pi}{6} + k\pi,\ k \in \mathbb{Z} \Leftrightarrow x = \frac{\pi}{3} + \frac{k\pi}{2},\ k \in \mathbb{Z}$$

z) $\tan^3 x = \tan x$

$$\text{tg}^3\,x = \text{tg}\,x \Leftrightarrow \text{tg}^3\,x - \text{tg}\,x = 0 \Leftrightarrow \text{tg}\,x\,(\text{tg}^2\,x - 1) = 0 \Leftrightarrow \text{tg}\,x = 0\ \vee\ \text{tg}^2\,x - 1 = 0 \Leftrightarrow$$

$$\Leftrightarrow \text{tg}\,x = 0\ \vee\ \text{tg}\,x = \pm 1 \Leftrightarrow x = k\pi\ \vee\ x = \frac{\pi}{4} + k\pi\ \vee\ x = -\frac{\pi}{4} + k\pi,\ k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x = k\pi\ \vee\ x = \frac{\pi}{4} + \frac{k\pi}{2},\ k \in \mathbb{Z}$$

2. Para cada uma das seguintes equações, determina as soluções que pertencem ao intervalo $[-\pi\,, 2\pi[$

a) $\sqrt{8}\sin\left(2x + \frac{\pi}{3}\right) = \sqrt{6}$

a) $$\sqrt{8}\,\text{sen}\left(2x + \frac{\pi}{3}\right) = \sqrt{6} \Leftrightarrow \text{sen}\left(2x + \frac{\pi}{3}\right) = \frac{\sqrt{6}}{\sqrt{8}} \Leftrightarrow \text{sen}\left(2x + \frac{\pi}{3}\right) = \sqrt{\frac{3}{4}} \Leftrightarrow$$

$$\Leftrightarrow \text{sen}\left(2x + \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \Leftrightarrow 2x + \frac{\pi}{3} = \frac{\pi}{3} + 2k\pi\ \vee\ 2x + \frac{\pi}{3} = \frac{2\pi}{3} + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow 2x = 2k\pi\ \vee\ 2x = \frac{\pi}{3} + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow x = k\pi\ \vee\ x = \frac{\pi}{6} + k\pi,\ k \in \mathbb{Z}$$

$x = k\pi$

$k = 0 \rightarrow x = 0 \in [-\pi, 2\pi[$

$k = 1 \rightarrow x = \pi \in [-\pi, 2\pi[$

$k = 2 \rightarrow x = 2\pi \notin [-\pi, 2\pi[$

$k = -1 \rightarrow x = -\pi \in [-\pi, 2\pi[$

$k = -2 \rightarrow x = -2\pi \notin [-\pi, 2\pi[$

$x = \frac{\pi}{6} + k\pi$

$k = 0 \rightarrow x = \frac{\pi}{6} \in [-\pi, 2\pi[$

$k = 1 \rightarrow x = \frac{7\pi}{6} \in [-\pi, 2\pi[$

$k = 2 \rightarrow x = \frac{13\pi}{6} \notin [-\pi, 2\pi[$

$$k = -1 \rightarrow x = -\frac{5\pi}{6} \in [-\pi, 2\pi[$$

$$k = -2 \rightarrow x = -\frac{11\pi}{6} \notin [-\pi, 2\pi[$$

Soluções: $x = 0 \vee x = \pi \vee x = -\pi \vee x = \frac{\pi}{6} \vee x = \frac{7\pi}{6} \vee x = -\frac{5\pi}{6}$

b) $\frac{\sin x}{x} = 0$

$$\frac{\operatorname{sen} x}{x} = 0 \Leftrightarrow \operatorname{sen} x = 0 \wedge x \neq 0 \Leftrightarrow x = k\pi, k \in \mathbb{Z} \wedge x \neq 0$$

$x = k\pi$

$$k = 0 \rightarrow x = 0 \notin [-\pi, 2\pi[\backslash\{0\}$$

$$k = 1 \rightarrow x = \pi \in [-\pi, 2\pi[\backslash\{0\}$$

$$k = 2 \rightarrow x = 2\pi \notin [-\pi, 2\pi[\backslash\{0\}$$

$$k = -1 \rightarrow x = -\pi \in [-\pi, 2\pi[\backslash\{0\}$$

$$k = -2 \rightarrow x = -2\pi \notin [-\pi, 2\pi[\backslash\{0\}$$

Soluções: $x = \pi \vee x = -\pi$

c) $2 \sin^2 2x - 3 \sin 2x + 1 = 0$

$$2\operatorname{sen}^2(2x) - 3\operatorname{sen}(2x) + 1 = 0 \Leftrightarrow \operatorname{sen}(2x) = \frac{3 \pm \sqrt{(-3)^2 - 4\times 2\times 1}}{2\times 2} \Leftrightarrow$$

$$\Leftrightarrow \operatorname{sen}(2x) = \frac{3 \pm 1}{4} \Leftrightarrow \operatorname{sen}(2x) = 1 \vee \operatorname{sen}(2x) = \frac{1}{2} \Leftrightarrow$$

$$\Leftrightarrow 2x = \frac{\pi}{2} + 2k\pi \vee 2x = \frac{\pi}{6} + 2k\pi \vee 2x = \frac{5\pi}{6} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{4} + k\pi \vee x = \frac{\pi}{12} + k\pi \vee x = \frac{5\pi}{12} + k\pi, k \in \mathbb{Z}$$

$x = \frac{\pi}{4} + k\pi$

$$k = 0 \rightarrow x = \frac{\pi}{4} \in [-\pi, 2\pi[$$

$$k = 1 \rightarrow x = \frac{5\pi}{4} \in [-\pi, 2\pi[$$

$$k = 2 \rightarrow x = \frac{9\pi}{4} \notin [-\pi, 2\pi[$$

$$k = -1 \rightarrow x = -\frac{3\pi}{4} \in [-\pi, 2\pi[$$

$$k = -2 \rightarrow x = -\frac{7\pi}{4} \notin [-\pi, 2\pi[$$

$x = \frac{\pi}{12} + k\pi$

$$k = 0 \rightarrow x = \frac{\pi}{12} \in [-\pi, 2\pi[$$

$$k = 1 \rightarrow x = \frac{13\pi}{12} \in [-\pi, 2\pi[$$

$$k = 2 \rightarrow x = \frac{25\pi}{12} \notin [-\pi, 2\pi[$$

$$k = -1 \rightarrow x = -\frac{11\pi}{12} \in [-\pi, 2\pi[$$

$$k = -2 \rightarrow x = -\frac{23\pi}{12} \notin [-\pi, 2\pi[$$

$$x = \frac{5\pi}{12} + k\pi \qquad k = 0 \rightarrow x = \frac{5\pi}{12} \in [-\pi, 2\pi[$$

$$k = 1 \rightarrow x = \frac{17\pi}{12} \in [-\pi, 2\pi[$$

$$k = 2 \rightarrow x = \frac{29\pi}{12} \notin [-\pi, 2\pi[$$

$$k = -1 \rightarrow x = -\frac{7\pi}{12} \in [-\pi, 2\pi[$$

$$k = -2 \rightarrow x = -\frac{19\pi}{12} \notin [-\pi, 2\pi[$$

Soluções: $x = \frac{\pi}{4} \vee x = \frac{5\pi}{4} \vee x = -\frac{3\pi}{4} \vee x = \frac{\pi}{12} \vee x = \frac{13\pi}{12} \vee$
$\vee\ x = -\frac{11\pi}{12} \vee x = \frac{5\pi}{12} \vee x = \frac{17\pi}{12} \vee x = -\frac{7\pi}{12}$

3. Determina as soluções da condição $\sqrt[3]{2\cos(\pi x)} = \sqrt[6]{3}$ que pertencem ao intervalo $[-1, 3]$

$$\sqrt[3]{2\cos(\pi x)} = \sqrt[6]{3} \Leftrightarrow \sqrt[3]{2\cos(\pi x)} = \sqrt[3]{\sqrt{3}} \Leftrightarrow 2\cos(\pi x) = \sqrt{3} \Leftrightarrow \cos(\pi x) = \frac{\sqrt{3}}{2} \Leftrightarrow$$

$$\Leftrightarrow \pi x = \frac{\pi}{6} + 2k\pi \vee \pi x = -\frac{\pi}{6} + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{1}{6} + 2k \vee x = -\frac{1}{6} + 2k,\ k \in \mathbb{Z}$$

$$x = \frac{1}{6} + 2k \qquad k = 0 \rightarrow x = \frac{1}{6} \in [-1, 3]$$

$$k = 1 \rightarrow x = \frac{13}{6} \in [-1, 3]$$

$$k = 2 \rightarrow x = \frac{25}{6} \notin [-1, 3]$$

$$k = -1 \rightarrow x = -\frac{11}{6} \notin [-1, 3]$$

$$x = -\frac{1}{6} + 2k \qquad k = 0 \rightarrow x = -\frac{1}{6} \in [-1, 3]$$

$$k = 1 \rightarrow x = \frac{11}{6} \in [-1, 3]$$

$$k = 2 \rightarrow x = \frac{23}{6} \notin [-1, 3]$$

$$k = -1 \rightarrow x = -\frac{13}{6} \notin [-1, 3]$$

Soluções: $x = \frac{1}{6} \vee x = \frac{13}{6} \vee x = -\frac{1}{6} \vee x = \frac{11}{6}$

4. Para cada uma das seguintes equações, determina as soluções que pertencem ao intervalo $]-2\pi, \pi]$

a) $1-2\tan\left(\frac{x}{2}+\frac{\pi}{3}\right)=3$

$$1-2\,\text{tg}\left(\frac{x}{2}+\frac{\pi}{3}\right)=3 \Leftrightarrow \text{tg}\left(\frac{x}{2}+\frac{\pi}{3}\right)=-1 \Leftrightarrow \frac{x}{2}+\frac{\pi}{3}=-\frac{\pi}{4}+k\pi,\ k\in\mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow \frac{x}{2}=-\frac{7\pi}{12}+k\pi,\ k\in\mathbb{Z} \Leftrightarrow x=-\frac{7\pi}{6}+2k\pi,\ k\in\mathbb{Z}$$

Como $\frac{x}{2}+\frac{\pi}{3}\neq\frac{\pi}{2}+k\pi,\ k\in\mathbb{Z}$ vem que $x\neq\frac{\pi}{3}+2k\pi,\ k\in\mathbb{Z}$. No intervalo $]-2\pi,\pi]$ vem $x\neq\frac{\pi}{3}\wedge x\neq-\frac{5\pi}{3}$

$x=-\frac{7\pi}{6}+2k\pi$

$k=0 \rightarrow x=-\frac{7\pi}{6}\in\,]-2\pi,\pi]\setminus\left\{-\frac{5\pi}{3},\frac{\pi}{3}\right\}$

$k=1 \rightarrow x=\frac{5\pi}{6}\in\,]-2\pi,\pi]\setminus\left\{-\frac{5\pi}{3},\frac{\pi}{3}\right\}$

$k=2 \rightarrow x=\frac{17\pi}{6}\notin\,]-2\pi,\pi]\setminus\left\{-\frac{5\pi}{3},\frac{\pi}{3}\right\}$

$k=-1 \rightarrow x=-\frac{19\pi}{6}\notin\,]-2\pi,\pi]\setminus\left\{-\frac{5\pi}{3},\frac{\pi}{3}\right\}$

Soluções: $x=-\frac{7\pi}{6} \vee x=\frac{5\pi}{6}$

b) $\tan^2 x-\tan x+\sqrt{3}=\sqrt{3}\tan x$

$$\text{tg}^2 x-\text{tg}\,x+\sqrt{3}=\sqrt{3}\,\text{tg}\,x \Leftrightarrow \text{tg}^2 x-\left(1+\sqrt{3}\right)\text{tg}\,x+\sqrt{3}=0 \Leftrightarrow$$

$$\Leftrightarrow \text{tg}\,x=\frac{1+\sqrt{3}\pm\sqrt{\left(-1-\sqrt{3}\right)^2-4\times1\times\sqrt{3}}}{2} \Leftrightarrow \text{tg}\,x=\frac{1+\sqrt{3}\pm\sqrt{1+2\sqrt{3}+3-4\sqrt{3}}}{2} \Leftrightarrow$$

$$\Leftrightarrow \text{tg}\,x=\frac{1+\sqrt{3}\pm\sqrt{1-2\sqrt{3}+3}}{2} \Leftrightarrow \text{tg}\,x=\frac{1+\sqrt{3}\pm\sqrt{\left(\sqrt{3}-1\right)^2}}{2} \Leftrightarrow$$

$$\Leftrightarrow \text{tg}\,x=\frac{1+\sqrt{3}\pm\left(\sqrt{3}-1\right)}{2} \Leftrightarrow$$

$$\Leftrightarrow \text{tg}\,x=\frac{1+\sqrt{3}+\sqrt{3}-1}{2} \vee \text{tg}\,x=\frac{1+\sqrt{3}-\sqrt{3}+1}{2} \Leftrightarrow \text{tg}\,x=\sqrt{3} \vee \text{tg}\,x=1 \Leftrightarrow$$

$$\Leftrightarrow x=\frac{\pi}{3}+k\pi \vee x=\frac{\pi}{4}+k\pi,\ k\in\mathbb{Z}$$

Como $x\neq\frac{\pi}{2}+k\pi,\ k\in\mathbb{Z}$. No intervalo $]-2\pi,\pi]$ vem $x\neq-\frac{3\pi}{2}\wedge x\neq-\frac{\pi}{2}\wedge x\neq\frac{\pi}{2}$

Designemos por A o conjunto $]-2\pi,\pi]\setminus\left\{-\frac{3\pi}{2},-\frac{\pi}{2},\frac{\pi}{2}\right\}$

$x=\frac{\pi}{3}+k\pi$

$k=0 \rightarrow x=\frac{\pi}{3}\in A$

$k=1 \rightarrow x=\frac{4\pi}{3}\notin A$

$k=-1 \rightarrow x=-\frac{2\pi}{3}\in A$

$k=-2 \rightarrow x=-\frac{5\pi}{3}\in A$

$k=-3 \rightarrow x=-\frac{8\pi}{3}\notin A$

$x=\frac{\pi}{4}+k\pi$

$k=0 \rightarrow x=\frac{\pi}{4}\in A$

$k=1 \rightarrow x=\frac{5\pi}{4}\notin A$

$k=-1 \rightarrow x=-\frac{3\pi}{4}\in A$

$k=-2 \rightarrow x=-\frac{7\pi}{4}\in A$

$k=-3 \rightarrow x=-\frac{11\pi}{4}\notin A$

Soluções: $x=\frac{\pi}{3} \vee x=-\frac{2\pi}{3} \vee x=-\frac{5\pi}{3} \vee x=\frac{\pi}{4} \vee x=-\frac{3\pi}{4} \vee$
$\vee\ x=-\frac{7\pi}{4}$

c) $\frac{3-\tan^2 x}{2} = \frac{1}{\cos^2 x}$

$$\frac{3-\text{tg}^2\, x}{2} = \frac{1}{\cos^2 x} \Leftrightarrow \frac{3-\text{tg}^2\, x}{2} = \text{tg}^2\, x + 1 \Leftrightarrow 3 - \text{tg}^2\, x = 2\,\text{tg}^2\, x + 2 \Leftrightarrow 3\,\text{tg}^2\, x = 1 \Leftrightarrow$$

$$\Leftrightarrow \text{tg}^2\, x = \frac{1}{3} \Leftrightarrow \text{tg}\, x = \pm\sqrt{\frac{1}{3}} \Leftrightarrow \text{tg}\, x = \pm\frac{1}{\sqrt{3}} \Leftrightarrow \text{tg}\, x = \pm\frac{\sqrt{3}}{3} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{6} + k\pi \vee x = -\frac{\pi}{6} + k\pi,\ k \in \mathbb{Z}$$

Como $x \neq \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$.

No intervalo $]-2\pi, \pi]$ vem $x \neq -\frac{3\pi}{2} \wedge x \neq -\frac{\pi}{2} \wedge x \neq \frac{\pi}{2}$

Designemos por A o conjunto $]-2\pi, \pi]\setminus\left\{-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}\right\}$

$x = \frac{\pi}{6} + k\pi$

$k = 0 \rightarrow x = \frac{\pi}{6} \in A$

$k = 1 \rightarrow x = \frac{7\pi}{6} \notin A$

$k = -1 \rightarrow x = -\frac{5\pi}{6} \in A$

$k = -2 \rightarrow x = -\frac{11\pi}{6} \in A$

$k = -3 \rightarrow x = -\frac{17\pi}{6} \notin A$

$x = -\frac{\pi}{6} + k\pi$

$k = 0 \rightarrow x = -\frac{\pi}{6} \in A$

$k = 1 \rightarrow x = \frac{5\pi}{6} \in A$

$k = 2 \rightarrow x = \frac{11\pi}{6} \notin A$

$k = -1 \rightarrow x = -\frac{7\pi}{6} \in A$

$k = -2 \rightarrow x = -\frac{13\pi}{6} \notin A$

Soluções: $x = \frac{\pi}{6} \vee x = -\frac{5\pi}{6} \vee x = -\frac{11\pi}{6} \vee x = -\frac{\pi}{6} \vee x = \frac{5\pi}{6} \vee$

$\vee\ x = -\frac{7\pi}{6}$