1. Resolve em $\mathbb{R}$ as seguintes equações:

a) $3 + 4\sin x = 5$

b) $2 + \sqrt{8}\sin x = 0$

c) $\sqrt{3} + 2\sin 4x = \sqrt{12}$

d) $1 - \sin(-x) = 2$

e) $\frac{1-5\sin 3x}{3} = 2$

f) $1 + \sin 4x = \frac{3+\sin(-4x)}{3}$

g) $\left(1 - 2\sin\frac{x}{2}\right)\left(1 + \sqrt{2}\sin x\right) = 0$

h) $2\sin^2 x + \sqrt{3}\sin x = 0$

i) $5 + 4\sin^2\left(\frac{x}{4}\right) = 7$

j) $2\sin^2 6x + 3\sin 6x + 1 = 0$

k) $\sin(\pi - 4x) + \sin(\pi + 4x) + \cos\left(\frac{3\pi}{2} - 4x\right) + \sin(2\pi - 4x) = \sqrt{3}$

l) $-\sqrt{3} + 6\cos x = \sqrt{12}$

m) $5 + 4\cos x = 3$

n) $\sqrt{8} + 5\cos\frac{x}{2} = 3\left(\sqrt{2} + \cos\frac{x}{2}\right)$

o) $\cos^2 x - \cos(\pi + x) = 0$

p) $\frac{\sin^2 x}{1-\cos x} = 2$

q) $2\cos^3 x - \cos x = 0$

r) $\cos\left(5x - \frac{2\pi}{3}\right) = \cos\left(3x + \frac{4\pi}{3}\right)$

s) $\sin\left(\frac{\pi}{2} - 4x\right) - \cos(\pi - 4x) = 1 - 4\sin\left(\frac{3\pi}{2} + 4x\right)$

t) $1 + 5\tan x = 6$

u) $4 + \sqrt{3}\tan\frac{x}{3} = 7$

v) $3\tan\left(5x + \frac{\pi}{4}\right) + \sqrt{12} = \sqrt{3}$

w) $\tan^2 \frac{x}{2} = \tan \frac{x}{2}$

x) $\tan^2 2x - 3 = 0$

y) $3 \tan^2 x = 3 + 2\sqrt{3} \tan x$

z) $\tan^3 x = \tan x$

2. Para cada uma das seguintes equações, determina as soluções que pertencem ao intervalo $[-\pi , 2\pi[$

a) $\sqrt{8} \sin\left(2x + \frac{\pi}{3}\right) = \sqrt{6}$

b) $\frac{\sin x}{x} = 0$

c) $2 \sin^2 2x - 3 \sin 2x + 1 = 0$

3. Determina as soluções da condição $\sqrt[3]{2 \cos(\pi x)} = \sqrt[6]{3}$ que pertencem ao intervalo $[-1 , 3]$

4. Para cada uma das seguintes equações, determina as soluções que pertencem ao intervalo $]-2\pi , \pi]$

a) $1 - 2 \tan\left(\frac{x}{2} + \frac{\pi}{3}\right) = 3$

b) $\tan^2 x - \tan x + \sqrt{3} = \sqrt{3} \tan x$

c) $\frac{3 - \tan^2 x}{2} = \frac{1}{\cos^2 x}$

Soluções

1.

a) $x = \frac{\pi}{6} + 2k\pi \vee x = \frac{5\pi}{6} + 2k\pi\,, k \in \mathbb{Z}$

b) $x = -\frac{\pi}{4} + 2k\pi \vee x = \frac{5\pi}{4} + 2k\pi\,, k \in \mathbb{Z}$

c) $x = \frac{\pi}{12} + \frac{k\pi}{2} \vee x = \frac{\pi}{6} + \frac{k\pi}{2}\,, k \in \mathbb{Z}$

d) $x = \frac{\pi}{2} + 2k\pi\ ,\ k \in \mathbb{Z}$

e) $x = \frac{\pi}{2} + \frac{2k\pi}{3}\ ,\ k \in \mathbb{Z}$

f) $x = \frac{k\pi}{4}\ ,\ k \in \mathbb{Z}$

g) $x = \frac{\pi}{3} + 4k\pi \vee x = \frac{5\pi}{3} + 4k\pi \vee x = -\frac{\pi}{4} + 2k\pi \vee x = \frac{5\pi}{4} + 2k\pi\,,\ k \in \mathbb{Z}$

h) $x = k\pi \vee x = -\frac{\pi}{3} + 2k\pi \vee x = \frac{4\pi}{3} + 2k\pi\,,\ k \in \mathbb{Z}$

i) $x = \pi + 2k\pi\,, k \in \mathbb{Z}$

j) $x = -\frac{\pi}{36} + \frac{k\pi}{3} \vee x = \frac{7\pi}{36} + \frac{k\pi}{3} \vee x = -\frac{\pi}{12} + \frac{k\pi}{3}, k \in \mathbb{Z}$

k) $x = -\frac{\pi}{12} + \frac{k\pi}{2} \vee x = \frac{\pi}{3} + \frac{k\pi}{2}\,, k \in \mathbb{Z}$

l) $x = \frac{\pi}{6} + 2k\pi \vee x = -\frac{\pi}{6} + 2k\pi\ ,\ k \in \mathbb{Z}$

m) $x = \frac{2\pi}{3} + 2k\pi \vee x = \frac{4\pi}{3} + 2k\pi\ ,\ k \in \mathbb{Z}$

n) $x = \frac{\pi}{2} + 4k\pi \vee x = -\frac{\pi}{2} + 4k\pi\ ,\ k \in \mathbb{Z}$

o) $x = \frac{\pi}{2} + k\pi \vee x = \pi + 2k\pi\ ,\ k \in \mathbb{Z}$

p) Impossível em $\mathbb{R}$

q) $x = \frac{\pi}{2} + k\pi \vee x = \frac{\pi}{4} + \frac{k\pi}{2}\ ,\ k \in \mathbb{Z}$

r) $x = \pi + k\pi \vee x = -\frac{\pi}{12} + \frac{k\pi}{4}\ ,\ k \in \mathbb{Z}$

s) $x = \frac{\pi}{6} + \frac{k\pi}{2} \vee x = \frac{\pi}{3} + \frac{k\pi}{2}\,, k \in \mathbb{Z}$

t) $x = \frac{\pi}{4} + k\pi\ ,\ k \in \mathbb{Z}$

u) $x = \pi + 3k\pi\ ,\ k \in \mathbb{Z}$

v) $x = -\frac{\pi}{12} + \frac{k\pi}{5}\ ,\ k \in \mathbb{Z}$

w) $x = 2k\pi \vee x = \frac{\pi}{2} + 2k\pi\ ,\ k \in \mathbb{Z}$

x) $x = \pm\frac{\pi}{6} + \frac{k\pi}{2}\ ,\ k \in \mathbb{Z}$

y) $x = \frac{\pi}{3} + \frac{k\pi}{2}\ ,\ k \in \mathbb{Z}$

z) $x = k\pi \vee x = \frac{\pi}{4} + \frac{k\pi}{2}\ ,\ k \in \mathbb{Z}$

2.

a) $x = 0 \vee x = \pi \vee x = -\pi \vee x = \frac{\pi}{6} \vee x = \frac{7\pi}{6} \vee x = -\frac{5\pi}{6}$

b) $x = \pi \vee x = -\pi$

c) $x = \frac{\pi}{4} \vee x = \frac{5\pi}{4} \vee x = -\frac{3\pi}{4} \vee x = \frac{\pi}{12} \vee x = \frac{13\pi}{12} \vee x = -\frac{11\pi}{12} \vee x = \frac{5\pi}{12} \vee x = \frac{17\pi}{12} \vee x = -\frac{7\pi}{12}$

3. $x = \frac{1}{6} \vee x = \frac{13}{6} \vee x = -\frac{1}{6} \vee x = \frac{11}{6}$

4.

a) $x = -\frac{7\pi}{6} \vee x = \frac{5\pi}{6}$

b) $x = \frac{\pi}{3} \vee x = -\frac{2\pi}{3} \vee x = -\frac{5\pi}{3} \vee x = \frac{\pi}{4} \vee x = -\frac{3\pi}{4} \vee x = -\frac{7\pi}{4}$

c) $x = \frac{\pi}{6} \vee x = -\frac{5\pi}{6} \vee x = -\frac{11\pi}{6} \vee x = -\frac{\pi}{6} \vee x = \frac{5\pi}{6} \vee x = -\frac{7\pi}{6}$