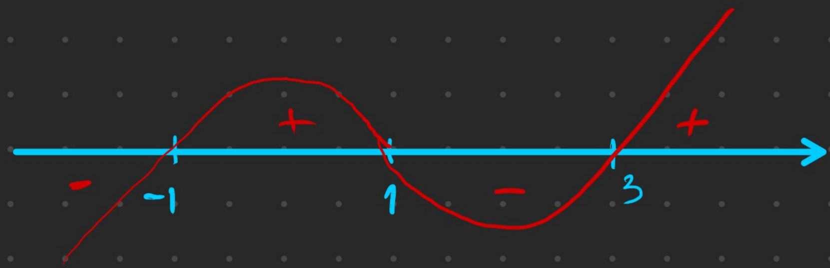
1.1 $2(x-1)(x+1)(x-3) \geq 0$



$x \in [-1, 1] \cup [3, +\infty[$

1.2 $f(x) < g(x) \Leftrightarrow 2(x+1)(x-1)(x-3) < x^4 - 1$

$\Leftrightarrow 2(x+1)(x-1)(x-3) < (x^2-1)(x^2+1)$

$\Leftrightarrow 2(x+1)(x-1)(x-3) < (x-1)(x+1)(x^2+1)$

$\Leftrightarrow 2\boxed{(x+1)(x-1)}(x-3) - \boxed{(x-1)(x+1)}(x^2+1) < 0$

$\Leftrightarrow (x+1)(x-1)\left[2(x-3) - (x^2+1)\right] < 0$

$\Leftrightarrow (x+1)(x-1)(2x - 6 - x^2 - 1) < 0$

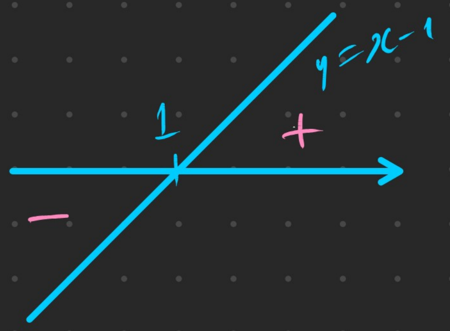
$\Leftrightarrow (x+1)(x-1)(-x^2 + 2x - 7) < 0$

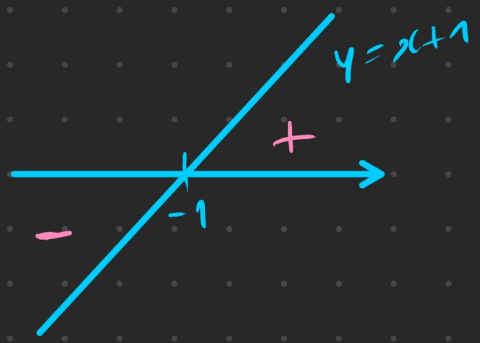
C.A.

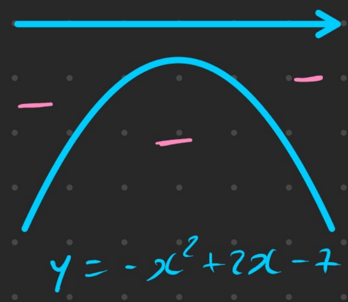
$-x^2 + 2x - 7 = 0 \Leftrightarrow x = \frac{-2 \pm \sqrt{4-28}}{-2}$ Impossível em $\mathbb{R}$

logo $-x^2 - 2x - 7$ não tem zeros e é sempre negativo.

	$-\infty$	-1		1	$+\infty$
$(x-1)$	$-$	$-$	$-$	0	$+$
$(x+1)$	$-$	0	$+$	$+$	$+$
$-x^2+2x-7$	$-$	$-$	$-$	$-$	$-$
$f(x)-g(x)<0$	$-$	0	$+$	0	$-$







$x \in]-1, 1[$

2.1 $f(x) = x(x+2)^2(x-2)$

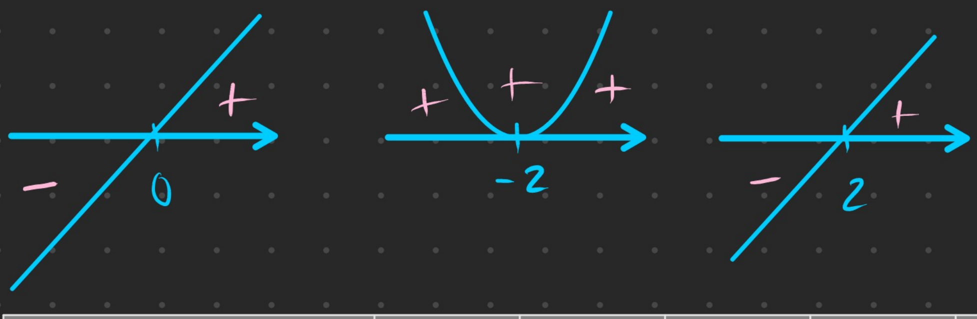
C.A.

$x(x+2)^2(x-2) = 0$

$\Leftrightarrow x=0 \vee (x+2)^2 = 0 \vee x-2=0$

$\Leftrightarrow x=0 \vee x+2=0 \vee x=2$

$\Leftrightarrow x=0 \vee x=-2 \vee x=2$



	$-\infty$	-2		0		2	$+\infty$
x	−	−	−	0	+	+	+
$(x+2)^2$	+	0	+	+	+	+	+
$x-2$	−	−	−	−	−	0	+
$f(x)$	+	0	+	0	−	0	+

f é positiva em $]-\infty, -2[\cup]-2, 0[\cup]2, +\infty[$

f é negativa em $]0, 2[$

2.2 $g(x) = 3x^3 - x^2 - 2x$

$$3x^3 - x^2 - 2x = 0 \Leftrightarrow x(3x^2 - x - 2) = 0$$
$$\Leftrightarrow x = 0 \vee 3x^2 - x - 2 = 0$$
$$\Leftrightarrow x = 0 \vee x = \frac{1 \pm \sqrt{1+24}}{6}$$
$$\Leftrightarrow x = 0 \vee x = 1 \vee x = -\frac{2}{3}$$

Assim, $g(x) = 3x(x-1)(x+\frac{2}{3})$

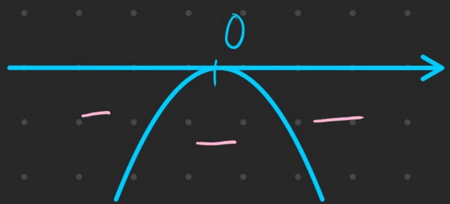
	$-\infty$	$-\frac{2}{3}$		0		1	$+\infty$
$3x$	$-$	$-$	$-$	0	$+$	$+$	$+$
$x-1$	$-$	$-$	$-$	$-$	$-$	0	$+$
$x+\frac{2}{3}$	$-$	0	$+$	$+$	$+$	$+$	$+$
$g(x)$	$-$	0	$+$	0	$-$	0	$+$

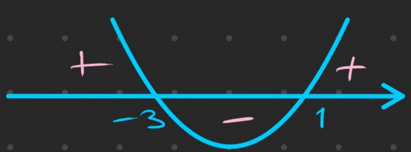
$g(x)$ é positiva em $]-\frac{2}{3}, 0[\cup]1, +\infty[$

$g(x)$ é negativa em $]-\infty, -\frac{2}{3}[\cup]0, 1[$

2.3 $h(x) = -x^4 - 2x^3 + 3x^2 = -x^2(x^2 + 2x - 3)$

$$-x^2(x^2 + 2x - 3) = 0 \Leftrightarrow -x^2 = 0 \vee x^2 + 2x - 3 = 0$$
$$\Leftrightarrow x = 0 \vee x = \frac{-2 \pm \sqrt{4+12}}{2}$$
$$\Leftrightarrow x = 0 \vee x = -3 \vee x = 1$$





	$-\infty$	-3		0		1	$+\infty$
$-x^2$	$-$	$-$	$-$	0	$-$	$-$	$-$
x^2+2x-3	$+$	0	$-$	$-$	$-$	0	$+$
$h(x)$	$-$	0	$+$	0	$+$	0	$-$

$h(x)$ é positiva em $]-3, 0[\cup]0, 1[$

$h(x)$ é negativa em $]-\infty, -3[\cup]1, +\infty[$

2.4) $i(x) = -x^3 + x^2 - 3x + 3$

$$\begin{array}{c|cccc} & -1 & 1 & -3 & 3 \\ 1 & & -1 & 0 & -3 \\ \hline & -1 & 0 & -3 & \boxed{0} \end{array}$$

$i(x) = (x-1)(-x^2-3) = -(x-1)(x^2+3)$

$-(x-1)(x^2+3) = 0 \Leftrightarrow -(x-1) = 0 \vee x^2+3=0$

$\Leftrightarrow x = 1 \vee x^2+3=0$

$x^2+3>0, \forall x \in \mathbb{R}$

	$-\infty$	1	$+\infty$
$-x+1$	$+$	0	$-$
x^2+3	$+$	$+$	$+$
$i(x)$	$+$	0	$-$

$i(x)$ é positiva em $]-\infty, 1[$

$i(x)$ é negativa em $]1, +\infty[$

3.1 $f(x) = 2x^3 + 3x^2 - 12x + k$

Como -2 é um zero duplo

$$
\begin{array}{r|rrrr}
 & 2 & 3 & -12 & k \\
-2 & & -4 & 2 & 20 \\
\hline
 & 2 & -1 & -10 & \boxed{k+20}
\end{array}
$$

Para que -2 seja um zero $k + 20 = 0$

$k = -20$ c.q.m.

3.2

$$
\begin{array}{r|rrrr}
 & 2 & 3 & -12 & -20 \\
-2 & & -4 & 2 & 20 \\
\hline
 & 2 & -1 & -10 & \boxed{0} \\
-2 & & -4 & 10 & \\
\hline
 & 2 & -5 & \boxed{0} &
\end{array}
$$

$$f(x) = (x+2)^2(2x-5) = 2(x+2)^2\left(x - \frac{5}{2}\right)$$

3.3

	-3		-2		$\frac{5}{2}$		5
$(x+2)^2$	nd	$+$	0	$+$	$+$	$+$	$+$
$x - \frac{5}{2}$	nd	$-$	$-$	$-$	0	$+$	$+$
$f(x)$	nd	$-$	0	$-$	0	$+$	$+$

f é negativa em $]-3, -2[\cup]-2, \frac{5}{2}[$

f é positiva em $]\frac{5}{2}, +\infty[$

3.4 Através do gráfico da função:

$f(5)$ é máximo da função $f(x)$

$f(5) = 2\times 5^3 + 3\times 5^2 - 12\times 5 - 20 = 245$

$f(-2) = 0$ é um máximo relativo

$f(1) = 2\times 1^3 + 3\times 1^2 - 12\times 1 - 20 = -27$ é mínimo absoluto

	-3		-2		1		5
$f(x)$	m.d.	$\nearrow$	0	$\searrow$	-27	$\nearrow$	245

f é crescente em $]-3,-2]$ e em $[1,5]$

f é decrescente em $[-2,1]$

3.5 $D'_f = [-27, 245]$

4. $D_f = [-2,2]$ $\quad f(x) = x^4 - x^2 - 2$

4.1 $x^4 - x^2 - 2 = 0 \underset{y=x^2}{\Leftrightarrow} y^2 - y - 2 = 0$

$\Leftrightarrow y = \dfrac{1 \pm \sqrt{1+8}}{2}$

$\Leftrightarrow y = 2 \vee y = -1$

$\underset{y=x^2}{\Leftrightarrow} x^2 = 2 \vee \underbrace{x^2 = -1}_{\text{Impossível em } \mathbb{R}}$

$\Leftrightarrow x = \pm\sqrt{2}$

$$
\begin{array}{c|ccccc}
 & 1 & 0 & -1 & 0 & -2 \\
\sqrt{2} & & \sqrt{2} & 2 & \sqrt{2} & 2 \\
\hline
 & 1 & \sqrt{2} & 1 & \sqrt{2} & \boxed{0} \\
-\sqrt{2} & & -\sqrt{2} & 0 & -\sqrt{2} & \\
\hline
 & 1 & 0 & 1 & \boxed{0} &
\end{array}
$$

Assim, $f(x) = (x-\sqrt{2})(x+\sqrt{2})(x^2+1)$

4.2 Pelo enunciado, o gráfico de f é simétrico relativamente a O_y, logo $f\left(\frac{\sqrt{2}}{2}\right)$ também é mínimo de $f(x)$.

$$f\left(\frac{\sqrt{2}}{2}\right) = f\left(-\frac{\sqrt{2}}{2}\right) = \left(\frac{\sqrt{2}}{2}\right)^4 - \left(\frac{\sqrt{2}}{2}\right)^2 - 2 = -\frac{9}{4}$$

$$f(-2) = f(2) = 2^4 - 2^2 - 2 = 10$$

$f(0) = 0^4 - 0^2 - 2 = -2$ é um máximo relativo

	-2		$-\frac{\sqrt{2}}{2}$		0		$\frac{\sqrt{2}}{2}$		2
$f(x)$	10	$\searrow$	$-\frac{9}{4}$	$\nearrow$	-2	$\searrow$	$-\frac{9}{4}$	$\nearrow$	10

f é decrescente em $\left[-2, -\frac{\sqrt{2}}{2}\right]$ e em $\left[0, \frac{\sqrt{2}}{2}\right]$

f é crescente em $\left[-\frac{\sqrt{2}}{2}, 0\right]$ e em $\left[\frac{\sqrt{2}}{2}, 2\right]$

$D'_f = \left[-\frac{9}{4}, 10\right]$

5.1 $f(x) = \dfrac{1}{x-3}$

$D_f = \{x \in \mathbb{R} : x - 3 \neq 0\} = \mathbb{R} \setminus \{3\}$

5.2 $f(x) = \dfrac{2x}{x^2-4}$

$D_f = \{x \in \mathbb{R} : x^2 - 4 \neq 0\} = \mathbb{R} \setminus \{-2, 2\}$

$x^2 - 4 = 0 \Leftrightarrow x^2 = 4 \Leftrightarrow x = \pm 2$

5.3 $f(x) = \dfrac{4x-5}{x^2+2x+1}$

$D_f = \{x \in \mathbb{R} : x^2 + 2x + 1 \neq 0\} = \mathbb{R} \setminus \{-1\}$

$x^2 + 2x + 1 = 0 \Leftrightarrow x = \dfrac{-2 \pm \sqrt{4-4}}{2} \Leftrightarrow x = -1$

5.4 $f(x) = \dfrac{1}{x} + \dfrac{1}{x+1}$

$D_f = \{x \in \mathbb{R} : x \neq 0 \wedge x + 1 \neq 0\} = \mathbb{R} \setminus \{-1, 0\}$

5.5 $f(x) = \dfrac{2x}{x^3 - 2x^2 + x}$

$D_f = \{x \in \mathbb{R} : x^3 - 2x^2 + x \neq 0\} = \mathbb{R} \setminus \{0, 1\}$

$x^3 - 2x^2 + x = 0 \Leftrightarrow x(x^2 - 2x + 1) = 0$

$\Leftrightarrow x = 0 \vee x^2 - 2x + 1 = 0$

$\Leftrightarrow x = 0 \vee (x-1)^2 = 0$

$\Leftrightarrow x = 0 \vee x = 1$

6.1 $f(x)=\dfrac{x}{x-5}$

$D_f=\{x\in\mathbb{R}: x-5\neq 0\}=\mathbb{R}\setminus\{5\}$

$f(x)=0 \Leftrightarrow \dfrac{x}{x-5}=0 \Leftrightarrow x=0 \wedge x\in D_f$

$C.S=\{0\}$

	$-\infty$	0		5	$+\infty$
x	$-$	0	$+$	$+$	$+$
$x-5$	$-$	$-$	$-$	0	$+$
$f(x)$	$+$	0	$-$	n.d	$+$

$f(x)>0 \Leftrightarrow x\in\,]-\infty,0[\,\cup\,]5,+\infty[$

$f(x)<0 \Leftrightarrow x\in\,]0,5[$

6.2 $f(x)=\dfrac{x+1}{x^2+4x+3}$

$D_f=\{x\in\mathbb{R}: x^2+4x+3\neq 0\}=\mathbb{R}\setminus\{-3,-1\}$

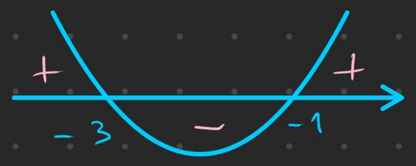
C.A.
$x^2+4x+3=0 \Leftrightarrow x=\dfrac{-4\pm\sqrt{16-12}}{2}$

$\Leftrightarrow x=-3 \vee x=-1$

$f(x)=0 \Leftrightarrow \dfrac{x+1}{x^2+4x+3}=0 \Leftrightarrow x+1=0 \wedge x\in D_f$

$\Leftrightarrow x=-1 \wedge x\in D_f$

não existem zeros



x^2+4x+3

	$-\infty$	-3		-1	$+\infty$
$x+1$	$-$	$-$	$-$	0	$+$
x^2+4x+3	$+$	0	$-$	0	$+$
$f(x)$	$-$	n.d.	$+$	n.d.	$+$

$$f(x) > 0 \Leftrightarrow x \in \,]-3, -1[$$

$$f(x) < 0 \Leftrightarrow x \in \,]-\infty, -3[\, \cup \,]-1, +\infty[$$

6.3 $$f(x) = \frac{x-1}{2x+1} + \frac{1}{x+1}$$

$$D_f = \{x \in \mathbb{R} : 2x+1 \neq 0 \land x+1 \neq 0\} = \mathbb{R} \setminus \{-1, -\tfrac{1}{2}\}$$

$$f(x) = 0 \Leftrightarrow \frac{x-1}{2x+1} + \frac{1}{x+1} = 0$$

$$\Leftrightarrow \frac{(x-1)(x+1) + 2x+1}{(2x+1)(x+1)} = 0$$

$$\Leftrightarrow x^2 - 1 + 2x + 1 = 0 \land x \in D_f$$

$$\Leftrightarrow x^2 + 2x = 0 \land x \in D_f$$

$$\Leftrightarrow x(x+2) = 0 \land x \in D_f$$

$$\Leftrightarrow x = 0 \lor x = -2 \land x \in D_f$$

$$C.S = \{-2, 0\}$$

	$-\infty$	-2		-1		$-\frac{1}{2}$		0	$+\infty$
x	$-$	$-$	$-$	$-$	$-$	$-$	$-$	0	$+$
$x+2$	$-$	0	$+$	$+$	$+$	$+$	$+$	$+$	$+$
$2x+1$	$-$	$-$	$-$	$-$	$-$	0	$+$	$+$	$+$
$x+1$	$-$	$-$	$-$	0	$+$	$+$	$+$	$+$	$+$
$f(x)$	$+$	0	$-$	nd	$+$	nd	$-$	0	$+$

$$f(x) > 0 \Leftrightarrow x \in]-\infty, -2[\cup]-1, -\tfrac{1}{2}[\cup]0, +\infty[$$

$$f(x) < 0 \Leftrightarrow x \in]-2, -1[\cup]-\tfrac{1}{2}, 0[$$

6.4

$$f(x) = \frac{x^2+2x+1}{x-2} \times \frac{x-3}{x^2-3x+2}$$

$$D_f = \{x \in \mathbb{R} : x-2 \neq 0 \wedge x^2-3x+2 \neq 0\} = \mathbb{R} \setminus \{1, 2\}$$

C.A.

$$x^2 - 3x + 2 = 0 \Leftrightarrow x = \frac{3 \pm \sqrt{9-8}}{2} \quad \frac{3+1}{2} = 2 \quad \frac{3-1}{2} = 1$$

$$f(x) = 0 \Leftrightarrow \frac{x^2+2x+1}{x-2} \times \frac{x-3}{x^2-3x+2} = 0$$

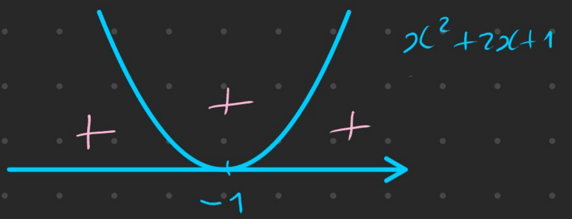
$$\Leftrightarrow (x^2+2x+1)(x-3) = 0 \wedge x \in D_f$$

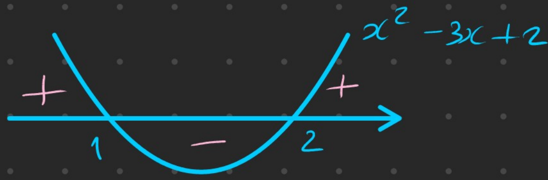
$$\Leftrightarrow x^2+2x+1 = 0 \wedge x = 3 \wedge x \in D_f$$

$$\Leftrightarrow (x+1)^2 = 0 \wedge x = 3 \wedge x \in D_f$$

$$\Leftrightarrow x = -1 \wedge x = 3 \wedge x \in D_f$$

C.S. $= \{-1, 3\}$





	$-\infty$	-1		1		2		3	$+\infty$
x^2+2x+1	+	0	+	+	+	+	+	+	+
$x-3$	−	−	−	−	−	−	−	0	+
$x-2$	−	−	−	−	−	0	+	+	+
x^2-3x+2	+	+	+	0	−	0	+	+	+
$f(x)$	+	0	+	nd	−	nd	−	0	+

$$f(x)>0 \Leftrightarrow x\in\,]-\infty,-1[\,\cup\,]-1,1[\,\cup\,]3,+\infty[$$

$$f(x)<0 \Leftrightarrow x\in\,]1,2[\,\cup\,]2,3[$$

6.5 $$f(x)=\frac{-x^3+x^2+2x-2}{x^2-3}$$

$$D_f=\{x\in\mathbb{R}: x^2-3\neq 0\}=\mathbb{R}\setminus\{-\sqrt{3},\sqrt{3}\}$$

$$f(x)=0 \Leftrightarrow \frac{-x^3+x^2+2x-2}{x^2-3}=0$$

$$\Leftrightarrow -x^3+x^2+2x-2=0 \;\wedge\; x\in D_f$$

$$\begin{array}{c|cccc} & -1 & 1 & 2 & -2 \\ 1 & & -1 & 0 & -2 \\ \hline & -1 & 0 & 2 & \underline{|\,0} \end{array}$$

$\Leftrightarrow (x-1)(-x^2+2)=0 \wedge x \in D_f$

$\Leftrightarrow x=1 \vee -x^2=-2 \wedge x \in D_f$

$\Leftrightarrow x=1 \vee x=-\sqrt{2} \vee x=\sqrt{2} \wedge x \in D_f$

$C.S = \{-\sqrt{2}, 1, \sqrt{2}\}$

	$-\infty$	$-\sqrt{3}$		$-\sqrt{2}$		1		$\sqrt{2}$		$\sqrt{3}$	0
$x+1$	$-$	$-$	$-$	$-$	$-$	0	$+$	$+$	$+$	$+$	$+$
$-x^2+2$	$-$	$-$	$-$	0	$+$	$+$	$+$	0	$-$	$-$	$-$
x^2-3	$+$	0	$-$	$-$	$-$	$-$	$-$	$-$	$-$	0	$+$
$f(x)$	$+$	nd	$-$	0	$+$	0	$-$	0	$+$	nd	$-$

$f(x)>0 \Leftrightarrow x \in]-\infty,-\sqrt{3}[\cup]-\sqrt{2},1[\cup]\sqrt{2},\sqrt{3}[$

$f(x)<0 \Leftrightarrow x \in]-\sqrt{3},-\sqrt{2}[\cup]1,\sqrt{2}[\cup]\sqrt{3},+\infty[$

6.6 $\quad f(x) = \dfrac{x^2-7}{x-2} - \dfrac{6}{x}$

$$D_f = \{x \in \mathbb{R} : x-2 \neq 0 \wedge x \neq 0\} = \mathbb{R} \setminus \{0, 2\}$$

$$f(x) = 0 \Leftrightarrow \frac{x-7}{x-2} - \frac{6}{x} = 0 \Leftrightarrow \frac{x^3 - 7x - 6x + 12}{x(x-2)} = 0$$

$$\Leftrightarrow x^3 - 13x + 12 = 0 \wedge x \in D_f$$

$$\begin{array}{c|cccc} & 1 & 0 & -13 & 12 \\ 1 & & 1 & 1 & -12 \\ \hline & 1 & 1 & -12 & \boxed{0} \end{array}$$

$(x-1)(x^2+x-12)$

$$\Leftrightarrow (x-1)(x^2+x-12) = 0 \wedge x \in D_f$$

$$\Leftrightarrow x = 1 \vee x = \frac{-1 \pm \sqrt{1+48}}{2} \wedge x \in D_f$$

$$\Leftrightarrow x = 1 \vee x = -4 \vee x = 3 \wedge x \in D_f$$

$$C.S. = \{-4, 1, 3\}$$

	$-\infty$	-4		0		1		2		3	$+\infty$
$x-1$	−	−	−	−	−	0	+	+	+	+	+
x^2+x-12	+	0	−	−	−	−	−	−	−	0	+
x	−	−	−	0	+	+	+	+	+	+	+
$x-2$	−	−	−	−	−	−	−	0	+	+	+
$f(x)$	−	0	+	nd	−	0	+	nd	−	0	+

$$f(x) > 0 \Leftrightarrow x \in \,]-4, 0[\, \cup \,]1, 2[\, \cup \,]3, +\infty[$$

$$f(x) < 0 \Leftrightarrow x \in \,]-\infty, -4[\, \cup \,]0, 1[\, \cup \,]2, 3[$$

6.7 $\quad f(x) = \dfrac{3x^2-2x}{9x^3+9x^2-4x-4}$

$$D_f = \{x \in \mathbb{R}: 9x^3+9x^2-4x-4 \neq 0\} = \mathbb{R} \setminus \{-1, -\tfrac{2}{3}, \tfrac{2}{3}\}$$

C. A.

	9	9	−4	−4
−1		−9	0	4
	9	0	−4	0

$(x+1)(9x^2-4)=0$

$\Leftrightarrow x=-1 \vee 9x^2=4$

$\Leftrightarrow x=-1 \vee x=\pm\frac{2}{3}$

$$f(x)=0 \Leftrightarrow \frac{3x^2-2x}{9x^3+9x^2-4x-4}=0 \Leftrightarrow 3x^2-2x=0 \wedge x \in D_f$$

$\Leftrightarrow x(3x-2)=0 \wedge x \in D_f$

$\Leftrightarrow x=0 \vee x=\frac{2}{3} \wedge x \in D_f$

C.S. $=\{0\}$

	$-\infty$	-1		$-\frac{2}{3}$		0		$\frac{2}{3}$	$+\infty$
$3x-2$	−	−	−	−	−	−	−	0	+
x	−	−	−	−	−	0	+	+	+
$x+1$	−	0	+	+	+	+	+	+	+
$9x^2-4$	+	+	+	0	−	−	−	0	+
$f(x)$	−	nd	+	nd	−	0	+	nd	+

$f(x)>0 \Leftrightarrow x \in]-1, -\frac{2}{3}[\cup]0, \frac{2}{3}[\cup]\frac{2}{3}, +\infty[$

$f(x)<0 \Leftrightarrow x \in]-\infty, -1[\cup]-\frac{2}{3}, 0[$

6.8 $\quad f(x) = \dfrac{x^3+x}{x^3-2x^2+x}$

$$D_f = \{x \in \mathbb{R} : x^3-2x^2+x \neq 0\} = \mathbb{R} \setminus \{0, 1\}$$

C.A.

$$x^3-2x^2+x=0 \Leftrightarrow x(x^2-2x+1)=0$$

$$\Leftrightarrow x=0 \vee (x-1)^2=0$$

$$\Leftrightarrow x=0 \vee x=1$$

$$f(x)=0 \Leftrightarrow \frac{x^3+x}{x^3-2x^2+x}=0 \Leftrightarrow x^3+x=0 \wedge x \in D_f$$

$$\Leftrightarrow x(x^2+1)=0 \wedge x \in D_f$$

$$\Leftrightarrow x=0 \vee \underbrace{x^2+1=0}_{\substack{x^2+1>0 \\ \forall x \in D_f}} \wedge x \in D_f$$

f não tem zeros

	$-\infty$	0		1	$+\infty$
x	$-$	0	$+$	$+$	$+$
x^2+1	$+$	$+$	$+$	$+$	$+$
x	$-$	0	$+$	$+$	$+$
x^2-2x+1	$+$	$+$	$+$	0	$+$
$f(x)$	$+$	n.d.	$+$	n.d.	$+$

$$f(x)>0 \Leftrightarrow x \in \mathbb{R} \setminus \{0, 1\}$$

7.1 $f(x) = \frac{5}{x-2}$

$D_f = \{x \in \mathbb{R} : x - 2 \neq 0\} = \mathbb{R} \setminus \{2\}$

- Equação da assíntota vertical: $x = 2$
- Equação da assíntota horizontal: $y = 0$

7.2 $g(x) \quad 2 - \frac{5}{x+4}$

$D_g = \{x \in \mathbb{R} : x + 4 \neq 0\} = \mathbb{R} \setminus \{4\}$

- Equação da assíntota vertical: $x = 4$
- Equação da assíntota horizontal: $y = 2$

7.3 $h(x) = \frac{x-1}{x-3}$

C. A.

$$\begin{array}{r|l} x - 1 & x - 3 \\ \hline -x + 3 & 1 \\ \hline 2 & \end{array}$$

$h(x) = 1 + \frac{2}{x-3}$ $\quad D_h = \{x \in \mathbb{R} : x - 3 \neq 0\} = \mathbb{R} \setminus \{3\}$

- Equação da assíntota vertical: $x = 3$
- Equação da assíntota horizontal: $y = 1$

7.4 $f(x) = \frac{3+2x}{2-3x}$

C.A.

$$\begin{array}{r|l} 2x+3 & -3x+2 \\ -2x+\frac{4}{3} & -\frac{2}{3} \\ \hline \frac{13}{3} & \end{array}$$

$f(x) = -\frac{2}{3} + \frac{\frac{13}{3}}{2-3x}$

$D_f = \{x \in \mathbb{R}: 2-3x \neq 0\} = \mathbb{R} \setminus \{\frac{2}{3}\}$

- Equação da assíntota vertical: $x = \frac{2}{3}$
- Equação da assíntota horizontal: $y = -\frac{2}{3}$

8.1 $\frac{x^2-x-2}{x^2+2x} = 0 \Leftrightarrow$

$\Leftrightarrow x^2-x-2=0 \wedge x \in D$

$\Leftrightarrow x = \frac{1 \pm \sqrt{1+8}}{2} \wedge x \in D$

$\Leftrightarrow x = -1 \vee x = 2 \wedge x \in D$

$C.S = \{-1, 2\}$

$D = \{x \in \mathbb{R}: x^2+2x \neq 0\} =$

$= \mathbb{R} \setminus \{-2, 0\}$

C.A.

$x^2+2x = 0 \Leftrightarrow x(x+2) = 0$

$\Leftrightarrow x = 0 \vee x = -2$

8.2 $\frac{x}{x^2-4} = \frac{1}{2-x} \Leftrightarrow$

$\Leftrightarrow \frac{x}{x^2-4} - \frac{1}{2-x} = 0$

$D = \{x \in \mathbb{R}: x^2-4 \neq 0 \wedge 2-x \neq 0\} =$

$= \mathbb{R} \setminus \{-2, 2\}$

$\Leftrightarrow \frac{x}{(x-2)(x+2)} + \frac{1}{x-2} = 0 \Leftrightarrow \frac{x+x+2}{(x-2)(x+2)} = 0$

$\Leftrightarrow 2x+2 = 0 \wedge x \in D \Leftrightarrow x = -1 \wedge x \in D$

$C.S = \{-1\}$

8.3 $\dfrac{x+2}{2x-x^2}+\dfrac{x}{x-2}=2 \Leftrightarrow$

$D=\{x\in\mathbb{R}: 2x-x^2\neq 0 \wedge x-2\neq 0\}$

$=\mathbb{R}\setminus\{0;2\}$

C.A.

$2x-x^2=0 \wedge x=2$

$x(2-x)=0 \wedge x=2$

$x=0 \vee x=2 \wedge x=2$

$$\Leftrightarrow \frac{x+2}{x(2-x)}-\frac{x}{2-x}-2=0$$

$$\Leftrightarrow \frac{x+2-x^2-4x+2x^2}{x(2-x)}=0$$

$$\Leftrightarrow x^2-3x+2=0 \wedge x\in D$$

$$\Leftrightarrow x=\frac{3\pm\sqrt{9-8}}{2} \wedge x\in D$$

$$\Leftrightarrow x=1 \vee x=2 \wedge x\in D$$

C.S. $=\{1\}$

9.1 $\dfrac{4-x^2}{x^2-2x}\geq\dfrac{1}{x} \Leftrightarrow \dfrac{4-x^2}{x(x-2)}-\dfrac{1}{x}\geq 0$

$$\Leftrightarrow \frac{4-x^2-x+2}{x(x-2)}\geq 0 \Leftrightarrow \frac{-x^2-x+6}{x(x-2)}\geq 0$$

C.A.

$-x^2-x+6=0 \Leftrightarrow x=-3 \vee x=2$

$x(x-2)=0 \Leftrightarrow x=0 \vee x=2$

	$-\infty$	-3		0		2	$+\infty$
$-x^2-x+6$	$-$	0	$+$	$+$	$+$	0	$-$
x	$-$	$-$	$-$	0	$+$	$+$	$+$
$x-2$	$-$	$-$	$-$	$-$	$-$	0	$+$
	$-$	0	$+$	n.d.	$-$	n.d.	$-$

$x\in[-3, 0[$

9.2

$$2x < \frac{x+2}{x-1} \Leftrightarrow 2x - \frac{x+2}{x-1} < 0$$

$$\Leftrightarrow \frac{2x^2 - 2x - x - 2}{x-1} < 0 \Leftrightarrow \frac{2x^2 - 3x - 2}{x-1} < 0$$

C.A. $2x^2 - 3x - 2 = 0 \Leftrightarrow x = -\frac{1}{2} \vee x = 2$

$x - 1 = 0 \Leftrightarrow x = 1$

	$-\infty$	$-\frac{1}{2}$		1		2	$+\infty$
$2x^2-3x-2$	$+$	0	$-$	$-$	$-$	0	$+$
$x-1$	$-$	$-$	$-$	0	$+$	$+$	$+$
$\frac{2x^2-3x-2}{x-1}$	$-$	0	$+$	nd	$-$	0	$+$

$$x \in \left]-\infty, -\frac{1}{2}\right[\cup \left]1, 2\right[$$

9.3

$$\frac{3 - x^3}{x^2(x+1)^3} > 0$$

C.A.

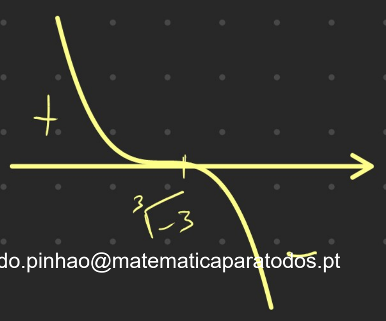
$3 - x^3 = 0 \Leftrightarrow x^3 = -3$

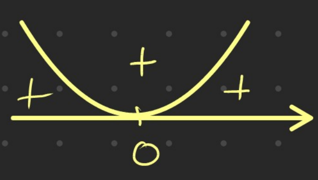
$\Leftrightarrow x = \sqrt[3]{-3}$

$x^2(x+1)^3 = 0 \Leftrightarrow x^2 = 0 \vee (x+1)^3 = 0$

$\Leftrightarrow x = 0 \vee x + 1 = 0$

$\Leftrightarrow x = 0 \vee x = -1$





	$-\infty$	$\sqrt[3]{-3}$		-1		0	$+\infty$
$3-x^3$	+	0	−	−	−	−	−
x^2	+	+	+	+	+	0	+
$(x+1)^3$	−	−	−	0	+	+	+
$\frac{3-x^3}{x^2(x+1)^3}$	−	0	+	nd	−	nd	−

$$x \in \left] \sqrt[3]{-3}, -1 \right[$$

9.4 $\frac{1}{x+1} + \frac{2x^2 - x - 1}{1 - x^2} \leq 0 \Leftrightarrow$

$\Leftrightarrow \frac{1}{x+1} + \frac{2x^2 - x - 1}{(1-x)(1+x)} \leq 0$

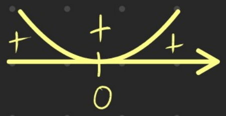
$\Leftrightarrow \frac{1 + x + 2x^2 - x - 1}{(1-x)(1+x)} \leq 0$

$\Leftrightarrow \frac{2x^2}{(1-x)(1+x)} \leq 0$

C. A.

$2x^2 = 0 \Leftrightarrow x = 0$

$(1-x)(1+x) = 0 \Leftrightarrow x = 1 \vee x = -1$



	$-\infty$	-1		0		1	$+\infty$
$2x^2$	$+$	$+$	$+$	0	$+$	$+$	$+$
$x+1$	$-$	0	$+$	$+$	$+$	$+$	$+$
$x-1$	$-$	$-$	$-$	$-$	$-$	0	$+$
$\dfrac{2x^2}{(x+1)(x-1)}$	$+$	nd	$-$	0	$-$	nd	$+$

$$\frac{1}{x+1}+\frac{2x^2-x-1}{1-x^2}\leq 0 \Leftrightarrow x\in\,]-1,1[$$

9.5 $$\frac{x-5}{x^2-x-2}\leq\frac{2}{x+1}\Leftrightarrow$$

C.A.

$x^2-x-2=0 \Leftrightarrow$

$x=-1 \vee x=2$

$$\Leftrightarrow \frac{x-5}{(x+1)(x-2)}-\frac{2}{x+1}\leq 0$$

$$\Leftrightarrow \frac{x-5-2x+4}{(x+1)(x-2)}\leq 0$$

$$\Leftrightarrow \frac{-x-1}{(x+1)(x-2)}\leq 0$$

x	$-\infty$	-1		2	$+\infty$
$-x-1$	$+$	0	$-$	$-$	$-$
$x+1$	$-$	0	$+$	$+$	$+$
$x-2$	$-$	$-$	$-$	0	$+$
$\dfrac{-x-1}{(x+1)(x-2)}$	$+$	nd	$+$	nd	$-$

$$x\in\,]2,+\infty[$$

9.6 $\dfrac{x^2-2}{x^2+3x} \geq \dfrac{2}{x} \Leftrightarrow \dfrac{x^2-2}{x(x+3)} - \dfrac{2}{x} \geq 0$

$$\Leftrightarrow \frac{x^2-2-2x-6}{x(x+3)} \geq 0 \Leftrightarrow \frac{x^2-2x-8}{x(x+3)} \geq 0$$

C.A.

$x^2-2x-8=0 \Leftrightarrow x=-2 \vee x=4$

$x(x+3)=0 \Leftrightarrow x=0 \vee x=-3$

	$-\infty$	-3		-2		0		4	$+\infty$
x^2-2x-8	+	+	+	0	−	−	−	0	+
x	−	−	−	−	−	0	+	+	+
$x+3$	−	0	+	+	+	+	+	+	+
$\dfrac{x^2-2x-8}{x(x+3)}$	+	nd	−	0	+	nd	−	0	+

$$\frac{x^2-2}{x(x+3)} - \frac{2}{x} \geq 0 \Leftrightarrow x \in \left]-\infty,-3\right[\cup \left[-2,0\right[\cup \left[4,+\infty\right[$$

9.7 $\dfrac{x^3-x^2}{x^2-1} > \dfrac{5}{x+1} \Leftrightarrow \dfrac{x^3-x^2}{(x-1)(x+1)} - \dfrac{5}{x+1} > 0$

$\Leftrightarrow \dfrac{x^3-x^2-5x+5}{(x-1)(x+1)} > 0$

$\Leftrightarrow \dfrac{(x-1)(x^2-5)}{(x-1)(x+1)} > 0$

C. A.

$$\begin{array}{r|rrrr} & 1 & -1 & -5 & 5 \\ 1 & & 1 & 0 & -5 \\ \hline & 1 & 0 & -5 & \boxed{0} \end{array}$$

$x^2-5=0 \Leftrightarrow$

$\Leftrightarrow x = \pm\sqrt{5}$

	$-\infty$	$-\sqrt{5}$		-1		1		$\sqrt{5}$	$+\infty$
$x-1$	$-$	$-$	$-$	$-$	$-$	0	$+$	$+$	$+$
x^2-5	$+$	0	$-$	$-$	$-$	$-$	$-$	0	$+$
$x-1$	$-$	$-$	$-$	$-$	$-$	0	$+$	$+$	$+$
$x+1$	$-$	$-$	$-$	0	$+$	$+$	$+$	$+$	$+$
$\dfrac{(x-1)(x^2-5)}{(x-1)(x+1)}$	$-$	0	$+$	nd	$-$	nd	$-$	0	$+$

$x \in \left]-\sqrt{5}, -1\right[\cup \left]\sqrt{5}, +\infty\right[$

9.2 $\quad 1 - \dfrac{1}{x+1} \geq \dfrac{4}{x^2+x} \Leftrightarrow \underset{(x^2+x)}{1} - \underset{(x)}{\dfrac{1}{x+1}} - \dfrac{4}{x(x+1)} \geq 0$

$\Leftrightarrow \dfrac{x^2+x-x-4}{x(x+1)} \geq 0 \Leftrightarrow \dfrac{x^2-4}{x(x+1)} \geq 0$

C.A.

$x^2-4=0 \Leftrightarrow x=-2 \vee x=2$

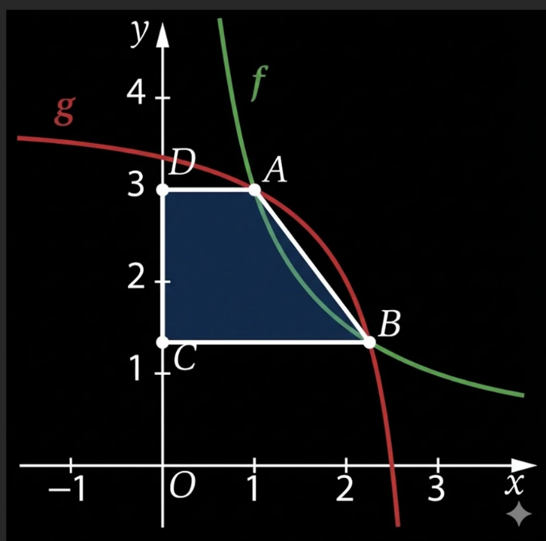
$x(x+1)=0 \Leftrightarrow x=0 \vee x=-1$

	$-\infty$	-2		-1		0		2	$+\infty$
x^2-4	+	0	−	−	−	−	−	0	+
x	−	−	−	−	−	0	+	+	+
$x+1$	−	−	−	0	+	+	+	+	+
$\dfrac{x^2-4}{x(x+1)}$	+	0	−	nd	+	nd	−	0	+

$x \in]-\infty, -2] \cup]-1, 0[\cup [2, +\infty[$

10. $\quad A = \dfrac{\overline{BC}+\overline{AD}}{2} \times \overline{CD}$

$P = \overline{AB} + \overline{BC} + \overline{CD} + \overline{DA}$



Os pontos A e B são a intersecção das funções $f(x)$ e $g(x)$

$$f(x) = g(x) \Leftrightarrow \frac{3}{x} = \frac{4x-10}{x-3} \Leftrightarrow \frac{3}{x} - \frac{4x-10}{x-3} = 0$$

$$\Leftrightarrow \frac{3x - 9 - 4x^2 + 10x}{x(x-3)} = 0$$

$$\Leftrightarrow -4x^2 + 13x - 9 = 0 \wedge x(x-3) \neq 0$$

$$\Leftrightarrow \left(x = 1 \vee x = \frac{9}{4}\right) \wedge (x \neq 0 \vee x \neq 3)$$

$$A(1, f(1)) = (1, 3)$$

$$B\left(\frac{9}{4}, f\left(\frac{9}{4}\right)\right) = \left(\frac{9}{4}, \frac{3}{\frac{9}{4}}\right) = \left(\frac{9}{4}, \frac{12}{9}\right) = \left(\frac{9}{4}, \frac{4}{3}\right)$$

$$\overline{AB} = \sqrt{\left(1 - \frac{9}{4}\right)^2 + \left(3 - \frac{4}{3}\right)^2} = \frac{25}{12}$$

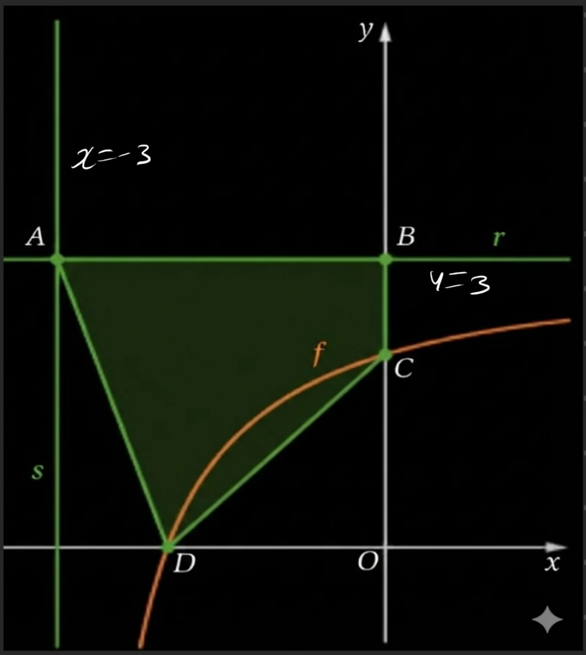
$$\overline{BC} = x_B = \frac{9}{4} \qquad \overline{DA} = x_A = 1$$

$$\overline{CD} = |y_D - y_C| = \left|3 - \frac{4}{3}\right| = \frac{5}{3}$$

$$A = \frac{\frac{9}{4} + 1}{2} \times \frac{5}{3} = \frac{65}{24}$$

$$P = \frac{25}{12} + \frac{9}{4} + \frac{5}{3} + 1 = 7$$

1.1.1 $f(x)=\frac{3x+6}{x+3}$



$A(-3,3)$ $B(0,3)$

$C(0,f(0))=\left(0,\frac{3\times 0+6}{0+3}\right)$

$=(0,2)$

$D(x,0)$

$f(x)=0 \Leftrightarrow \frac{3x+6}{x+3}=0$

$\Leftrightarrow 3x+6=0 \wedge x\neq -3$

$\Leftrightarrow x=-2 \wedge x\neq -3$

$A_{[ABCD]} = A_{\substack{\text{Trapézio}\\ [ABOD]}} - A_{\substack{\text{triângulo}\\ [CDO]}}$

$A_{[ABOD]} = \frac{\overline{AB}+\overline{OD}}{2}\times \overline{OB}$

C. A.

$\overline{AB}=|x_A|=|-3|=3$

$\overline{OD}=|x_D|=|-2|=2$

$\overline{OB}=|y_B|=|3|=3$

$=\frac{3+2}{2}\times 3$

$=\frac{15}{2}$

$A_{[OCD]}=\frac{\overline{OD}\times\overline{OC}}{2}=$

$=\frac{2\times 2}{2}=2$

C. A.

$\overline{OD}=2$

$\overline{OC}=|y_C|=|2|=2$

$A_{[ABCD]}=\frac{15}{2}-2=\frac{11}{2}$

11.2 $f(x)_{|\,]-3,+\infty[}(x) \geq 1 \Leftrightarrow \frac{3x+6}{x+3} \geq 1 \wedge x \in]-3,+\infty[$

$\Leftrightarrow \frac{3x+6}{x+3} - 1 \geq 0 \Leftrightarrow \frac{3x+6-x-3}{x+3} \geq 0$

$\Leftrightarrow \frac{2x+3}{x+3} \geq 0$

C. A.

$2x+3 = 0 \Leftrightarrow x = -\frac{3}{2}$

$x+3 = 0 \Leftrightarrow x = -3$

	-3		$-\frac{3}{2}$	$+\infty$
$2x+3$	n.d.	$-$	0	$+$
$x+3$	n.d.	$+$	$+$	$+$
$\frac{2x+3}{x+3}$	n.d.	$-$	0	$+$

$x \in \left[-\frac{3}{2}, +\infty\right[$