TRIGONOMETRIA – REDUÇÃO AO 1º QUADRANTE

RESOLUÇÃO

1. Simplifica o mais possível as seguintes expressões.

a) $\sin(\pi-\alpha)-\sin(2\pi-\alpha)+\cos(-\alpha)+\cos(\pi-\alpha)$

$\operatorname{sen}(\pi-\alpha)-\operatorname{sen}(2\pi-\alpha)+\cos(-\alpha)+\cos(\pi-\alpha)=$
$=\operatorname{sen}\alpha+\operatorname{sen}\alpha+\cos\alpha-\cos\alpha=2\operatorname{sen}\alpha$

b) $\tan(\pi-\alpha)-\tan(2\pi-\alpha)-3\sin(-\alpha)-2\sin(2\pi+\alpha)$

$\operatorname{tg}(\pi-\alpha)-\operatorname{tg}(2\pi-\alpha)-3\operatorname{sen}(-\alpha)-2\operatorname{sen}(2\pi+\alpha)=$
$=-\operatorname{tg}\alpha+\operatorname{tg}\alpha+3\operatorname{sen}\alpha-2\operatorname{sen}\alpha=\operatorname{sen}\alpha$

c) $\sin(-\alpha)\sin(\pi+\alpha)-\cos(-\alpha)\cos(\pi-\alpha)$

$\operatorname{sen}(-\alpha)\times\operatorname{sen}(\pi+\alpha)-\cos(-\alpha)\times\cos(\pi-\alpha)=$
$=-\operatorname{sen}\alpha\times(-\operatorname{sen}\alpha)-\cos\alpha\times(-\cos\alpha)=\operatorname{sen}^2\alpha+\cos^2\alpha=1$

d) $5\sin\left(\frac{\pi}{2}-\alpha\right)+\cos\left(\frac{\pi}{2}+\alpha\right)-\sin(\pi+\alpha)+3\cos(\pi+\alpha)$

$5\operatorname{sen}\left(\frac{\pi}{2}-\alpha\right)+\cos\left(\frac{\pi}{2}+\alpha\right)-\operatorname{sen}(\pi+\alpha)+3\cos(\pi+\alpha)=$
$=5\cos\alpha-\operatorname{sen}\alpha+\operatorname{sen}\alpha-3\cos\alpha=2\cos\alpha$

e) $\sin\left(\frac{3\pi}{2}-\alpha\right)+\cos(2\pi-\alpha)-\tan(-\alpha)\cos(2\pi-\alpha)+\cos\left(\frac{3\pi}{2}+\alpha\right)$

$\operatorname{sen}\left(\frac{3\pi}{2}-\alpha\right)+\cos(2\pi-\alpha)-\operatorname{tg}(-\alpha)\times\cos(2\pi+\alpha)+\cos\left(\frac{3\pi}{2}+\alpha\right)=$
$=-\cos\alpha+\cos\alpha+\operatorname{tg}\alpha\times\cos\alpha+\operatorname{sen}\alpha=\frac{\operatorname{sen}\alpha}{\cos\alpha}\times\cos\alpha+\operatorname{sen}\alpha=2\operatorname{sen}\alpha$

f) $\sin(\alpha-\pi)-\tan(\alpha+\pi)-\tan(2\pi-\alpha)-5\cos\left(\alpha-\frac{3\pi}{2}\right)$

$\operatorname{sen}(\alpha-\pi)-\operatorname{tg}(\alpha+\pi)-\operatorname{tg}(2\pi-\alpha)-5\cos\left(\alpha-\frac{3\pi}{2}\right)=$
$=-\operatorname{sen}\alpha-\operatorname{tg}\alpha+\operatorname{tg}\alpha+5\operatorname{sen}\alpha=4\operatorname{sen}\alpha$

g) $\sin(\alpha-5\pi)\tan(-\alpha-4\pi)-3\sin\left(\alpha-\frac{\pi}{2}\right)+2\cos(7\pi-\alpha)$

$\operatorname{sen}(\alpha-5\pi)\times\operatorname{tg}(-\alpha-4\pi)-3\operatorname{sen}\left(\alpha-\frac{\pi}{2}\right)+2\cos(7\pi-\alpha)=$
$=-\operatorname{sen}\alpha\times(-\operatorname{tg}\alpha)+3\cos\alpha-2\cos\alpha=\operatorname{sen}\alpha\times\frac{\operatorname{sen}\alpha}{\cos\alpha}+\cos\alpha=$
$=\frac{\operatorname{sen}^2\alpha}{\cos\alpha}+\cos\alpha=\frac{\operatorname{sen}^2\alpha+\cos^2\alpha}{\cos\alpha}=\frac{1}{\cos\alpha}$

h) $\cos\left(\frac{5\pi}{2}+\alpha\right)-\sin\left(\alpha-\frac{7\pi}{2}\right)-\cos(7\pi-\alpha)+2\sin(\alpha+4\pi)$

$\cos\left(\frac{5\pi}{2}+\alpha\right)-\operatorname{sen}\left(\alpha-\frac{7\pi}{2}\right)-\cos(7\pi-\alpha)+2\operatorname{sen}(\alpha+4\pi)=$
$=-\operatorname{sen}\alpha-\cos\alpha+\cos\alpha+2\operatorname{sen}\alpha=\operatorname{sen}\alpha$

2. Qual é o valor lógico da proposição:

Se $\sin(\pi - x) > 0$ e $\tan\left(-\frac{\pi}{2} - x\right) < 0$, então $\cos(-\pi - x) < 0$

$\operatorname{sen}(\pi - x) > 0 \Leftrightarrow \operatorname{sen} x > 0$

$\tan\left(-\frac{\pi}{2} - x\right) < 0 \Leftrightarrow \tan x < 0$ sabemos que a tangente é negativa nos 2.º e 4.º quadrantes.

Como $\operatorname{sen} x > 0$, então a tangente está no 2.º quadrante.

Temos que $\cos(-\pi - x) = \cos(-(\pi + x)) = \cos(\pi + x) = -\cos x$

No 2.º quadrante, o cosseno é negativo, então:

$-\cos(x) < 0 \Leftrightarrow \cos(-\pi - x) < 0$

Portanto a proposição é verdadeira.

3. Prova que, para qualquer *x* para o qual as expressões têm significado, se tem:

a) $1 - \frac{\cos(2\pi + x)}{\tan(2\pi + x)\sin x + \cos(2\pi - x)} = \sin^2 x$

$$1 - \frac{\cos(2\pi + x)}{\operatorname{tg}(2\pi + x) \times \operatorname{sen} x + \cos(2\pi - x)} = 1 - \frac{\cos x}{\operatorname{tg} x \times \operatorname{sen} x + \cos x} =$$

$$= 1 - \frac{\cos x}{\frac{\operatorname{sen} x}{\cos x} \times \operatorname{sen} x + \cos x} = 1 - \frac{\cos x}{\frac{\operatorname{sen}^2 x + \cos^2 x}{\cos x}} = 1 - \frac{\cos x}{\frac{1}{\cos x}} = 1 - \cos^2 x = \operatorname{sen}^2 x$$

b) $2\sin\left(\frac{\pi}{2} - x\right)\sin\left(\frac{\pi}{2} + x\right) - \cos(2\pi) = \left[1 - \sqrt{2}\sin(2\pi + x)\right]\left[1 - \sqrt{2}\sin(\pi + x)\right]$

$$2\operatorname{sen}\left(\frac{\pi}{2} - x\right)\operatorname{sen}\left(\frac{\pi}{2} + x\right) - \cos(2\pi) = 2\cos x\cos x - \cos(2\pi) = 2\cos^2 x - 1 =$$

$$= 2(1 - \operatorname{sen}^2 x) - 1 = 2 - 2\operatorname{sen}^2 x - 1 = 1 - 2\operatorname{sen}^2 x = 1 - \left(\sqrt{2}\operatorname{sen} x\right)^2 =$$

$$= \left(1 - \sqrt{2}\operatorname{sen} x\right)\left(1 + \sqrt{2}\operatorname{sen} x\right) = \left(1 - \sqrt{2}\operatorname{sen}(2\pi + x)\right)\left(1 - \sqrt{2}\operatorname{sen}(\pi + x)\right)$$

4. Determina o valor das seguintes expressões.

a) $\sin\left(-\frac{\pi}{6}\right) + \sqrt{6}\sin\left(-\frac{\pi}{4}\right)\sin\left(-\frac{\pi}{3}\right)$

$$\operatorname{sen}\left(-\frac{\pi}{6}\right) + \sqrt{6}\operatorname{sen}\left(-\frac{\pi}{4}\right)\operatorname{sen}\left(-\frac{\pi}{3}\right) =$$

$$= -\operatorname{sen}\left(\frac{\pi}{6}\right) + \left(-\sqrt{6}\operatorname{sen}\left(\frac{\pi}{4}\right)\right)\left(-\operatorname{sen}\left(\frac{\pi}{3}\right)\right) =$$

$$= -\operatorname{sen}\left(\frac{\pi}{6}\right) + \sqrt{6}\operatorname{sen}\left(\frac{\pi}{4}\right)\operatorname{sen}\left(\frac{\pi}{3}\right) =$$

$$= -\frac{1}{2} + \sqrt{6} \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} = -\frac{1}{2} + \frac{6}{4} = -\frac{1}{2} + \frac{3}{2} = 1$$

b) $\sin\left(-\frac{3\pi}{4}\right) - \cos\left(-\frac{\pi}{4}\right) + \sqrt{5 - \sqrt{3}\tan\left(-\frac{\pi}{3}\right)}$

$$\text{sen}\left(-\frac{3\pi}{4}\right) - \cos\left(-\frac{\pi}{4}\right) + \sqrt{5 - \sqrt{3}\,\text{tg}\left(-\frac{\pi}{3}\right)} =$$
$$= -\text{sen}\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{4}\right) + \sqrt{5 + \sqrt{3}\,\text{tg}\left(\frac{\pi}{3}\right)} =$$
$$= -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} + \sqrt{5 + \sqrt{3} \times \sqrt{3}} = -\sqrt{2} + \sqrt{8} = -\sqrt{2} + 2\sqrt{2} = \sqrt{2}$$

c) $\frac{\sin\frac{5\pi}{6} + \cos\left(-\frac{3\pi}{4}\right)}{2 + \cos(-\pi) + \cos\left(-\frac{2\pi}{3}\right) - \sin\left(-\frac{3\pi}{4}\right)}$

$$\frac{\text{sen}\,\frac{5\pi}{6} + \cos\left(-\frac{3\pi}{4}\right)}{2 + \cos(-\pi) + \cos\left(-\frac{2\pi}{3}\right) - \text{sen}\left(-\frac{3\pi}{4}\right)} = \frac{\text{sen}\,\frac{\pi}{6} - \cos\frac{\pi}{4}}{2 - 1 - \cos\frac{\pi}{3} + \text{sen}\,\frac{\pi}{4}} =$$
$$= \frac{\frac{1}{2} - \frac{\sqrt{2}}{2}}{1 - \frac{1}{2} + \frac{\sqrt{2}}{2}} = \frac{\frac{1-\sqrt{2}}{2}}{\frac{1+\sqrt{2}}{2}} = \frac{1-\sqrt{2}}{1+\sqrt{2}} = \frac{\left(1-\sqrt{2}\right)\left(1-\sqrt{2}\right)}{\left(1+\sqrt{2}\right)\left(1-\sqrt{2}\right)} = \frac{1 - 2\sqrt{2} + 2}{1 - 2} =$$
$$= \frac{3 - 2\sqrt{2}}{-1} = 2\sqrt{2} - 3$$

d) $\frac{\sin\left(-\frac{17\pi}{3}\right)}{\cos\frac{17\pi}{4}} + \sqrt{6}\sin\frac{37\pi}{6}$

$$\frac{\text{sen}\left(-\frac{17\pi}{3}\right)}{\cos\frac{17\pi}{4}} + \sqrt{6}\,\text{sen}\,\frac{37\pi}{6} = \frac{\text{sen}\left(-\frac{17\pi}{3} + 6\pi\right)}{\cos\left(\frac{17\pi}{4} - 4\pi\right)} + \sqrt{6}\,\text{sen}\left(\frac{37\pi}{6} - 6\pi\right) =$$
$$= \frac{\text{sen}\,\frac{\pi}{3}}{\cos\frac{\pi}{4}} + \sqrt{6}\,\text{sen}\,\frac{\pi}{6} = \frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{2}}{2}} + \sqrt{6} \times \frac{1}{2} = \frac{\sqrt{3}}{\sqrt{2}} + \frac{\sqrt{6}}{2} = \frac{\sqrt{6}}{2} + \frac{\sqrt{6}}{2} = \sqrt{6}$$

5. Seja $x \in \left[\frac{3\pi}{2}, 2\pi\right]$, tal que $\cos\left(\frac{\pi}{2} - x\right)\tan(\pi + x) - \sin\left(\frac{3\pi}{2} - x\right) = 2$.

a) Determina o valor de x .

$$\cos\left(\frac{\pi}{2} - x\right) \times \operatorname{tg}(\pi + x) - \operatorname{sen}\left(\frac{3\pi}{2} - x\right) = 2 \ \wedge \ x \in \left[\frac{3\pi}{2}, 2\pi\right] \Leftrightarrow$$

$$\Leftrightarrow \operatorname{sen} x \times \operatorname{tg} x + \cos x = 2 \ \wedge \ x \in \left[\frac{3\pi}{2}, 2\pi\right] \Leftrightarrow$$

$$\Leftrightarrow \operatorname{sen} x \times \frac{\operatorname{sen} x}{\cos x} + \cos x = 2 \ \wedge \ x \in \left[\frac{3\pi}{2}, 2\pi\right] \Leftrightarrow$$

$$\Leftrightarrow \frac{\operatorname{sen}^2 x + \cos^2 x}{\cos x} = 2 \ \wedge \ x \in \left[\frac{3\pi}{2}, 2\pi\right] \Leftrightarrow \frac{1}{\cos x} = 2 \ \wedge \ x \in \left[\frac{3\pi}{2}, 2\pi\right] \Leftrightarrow$$

$$\Leftrightarrow 1 = 2\cos x \wedge \cos x \neq 0 \ \wedge \ x \in \left[\frac{3\pi}{2}, 2\pi\right] \Leftrightarrow$$

$$\Leftrightarrow \cos x = \frac{1}{2} \ \wedge \ x \in \left[\frac{3\pi}{2}, 2\pi\right] \Leftrightarrow x = \frac{5\pi}{3}$$

b) Determina o valor de $\tan(\pi - x) + 2\cos(\pi + x) - 2\sin(2\pi - x)$.

$$\operatorname{tg}\left(\pi - \frac{5\pi}{3}\right) + 2\cos\left(\pi + \frac{5\pi}{3}\right) - 2\operatorname{sen}\left(2\pi - \frac{5\pi}{3}\right) =$$
$$= \operatorname{tg}\left(-\frac{2\pi}{3}\right) + 2\cos\frac{8\pi}{3} - 2\operatorname{sen}\frac{\pi}{3} = \operatorname{tg}\frac{\pi}{3} + 2\cos\frac{2\pi}{3} - 2\operatorname{sen}\frac{\pi}{3} =$$
$$= \sqrt{3} - 2\cos\frac{\pi}{3} - 2 \times \frac{\sqrt{3}}{2} = \sqrt{3} - 2 \times \frac{1}{2} - \sqrt{3} = -1$$