1.1 $\cos\left(-\frac{4\pi}{3}\right) = \cos\left(\frac{4\pi}{3}\right) = \cos\left(\pi + \frac{\pi}{3}\right)$

$= -\cos\frac{\pi}{3} = -\frac{1}{2}$

1.2 $\operatorname{sen}\left(\frac{11\pi}{6}\right) = \operatorname{sen}\left(\frac{12\pi}{6} - \frac{\pi}{6}\right) = \operatorname{sen}\left(2\pi - \frac{\pi}{6}\right)$

$= \operatorname{sen}\left(-\frac{\pi}{6}\right) = -\operatorname{sen}\frac{\pi}{6} = -\frac{1}{2}$

1.3 $\tan\left(\frac{3\pi}{4}\right) = \tan\left(\pi - \frac{\pi}{4}\right) = -\tan\frac{\pi}{4} = -1$

1.4 $\operatorname{sen}\left(\frac{2\pi}{3}\right) + \cos\left(-\frac{7\pi}{6}\right) + \tan\left(\frac{7\pi}{4}\right) =$

$= \operatorname{sen}\left(\pi - \frac{\pi}{3}\right) + \cos\left(\frac{7\pi}{6}\right) + \tan\left(\frac{8\pi}{4} - \frac{\pi}{4}\right) =$

$= \operatorname{sen}\left(\frac{\pi}{3}\right) + \cos\left(\pi + \frac{\pi}{6}\right) + \tan\left(2\pi - \frac{\pi}{4}\right)$

$= \frac{\sqrt{3}}{2} - \cos\frac{\pi}{6} + \tan\left(-\frac{\pi}{4}\right) =$

$= \cancel{\frac{\sqrt{3}}{2}} - \cancel{\frac{\sqrt{3}}{2}} - \tan\frac{\pi}{4} = -1$

1.5 $\cos\left(\frac{5\pi}{4}\right) - \operatorname{sen}\left(-\frac{3\pi}{4}\right) - 2\tan\left(\frac{4\pi}{3}\right) =$

$= \cos\left(\pi + \frac{\pi}{4}\right) + \operatorname{sen}\left(\frac{3\pi}{4}\right) - 2\tan\left(\pi + \frac{\pi}{3}\right) =$

$= -\cos\frac{\pi}{4} + \operatorname{sen}\left(\pi - \frac{\pi}{4}\right) - 2\tan\frac{\pi}{3} =$

$= -\frac{\sqrt{2}}{2} + \operatorname{sen}\frac{\pi}{4} - 2\times\sqrt{3} = -\cancel{\frac{\sqrt{2}}{2}} + \cancel{\frac{\sqrt{2}}{2}} - 2\sqrt{3} =$

$= -2\sqrt{3}$

1.6 $\sqrt{2}\,\operatorname{sen}\left(\frac{7\pi}{4}\right) + \sqrt{3}\tan\left(\frac{4\pi}{3}\right) =$

$= \sqrt{2}\,\operatorname{sen}\left(\frac{8\pi}{4} - \frac{\pi}{4}\right) + \sqrt{3}\tan\left(\pi + \frac{\pi}{3}\right)$

$= \sqrt{2}\,\operatorname{sen}\left(-\frac{\pi}{4}\right) + \sqrt{3}\tan\frac{\pi}{3} =$

$= -\sqrt{2}\,\operatorname{sen}\frac{\pi}{4} + \sqrt{3} \times \sqrt{3} = -\sqrt{2} \times \frac{\sqrt{2}}{2} + 3$

$= -1 + 3 = 2$

1.7 $\dfrac{\cos\left(\frac{7\pi}{6}\right) - \operatorname{sen}\left(-\frac{5\pi}{3}\right)}{\tan\left(\frac{13\pi}{6}\right)} = \dfrac{\cos\left(\pi + \frac{\pi}{6}\right) + \operatorname{sen}\left(\frac{5\pi}{3}\right)}{\tan\left(\frac{12\pi}{6} + \frac{\pi}{6}\right)}$

$= \dfrac{-\cos\frac{\pi}{6} + \operatorname{sen}\left(\pi - \frac{\pi}{3}\right)}{\tan\frac{\pi}{6}} = \dfrac{-\frac{\sqrt{3}}{2} - \operatorname{sen}\frac{\pi}{3}}{\sqrt{3}}$

$= \dfrac{-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}}{\sqrt{3}} = \dfrac{-\frac{\cancel{2}\sqrt{3}}{\cancel{2}}}{\sqrt{3}} = -\dfrac{\sqrt{3}}{\sqrt{3}} = -1$

1.8 $\tan \pi + \operatorname{sen} \frac{3\pi}{2} - 2\cos\left(\frac{7\pi}{4}\right) - \operatorname{sen}\left(-\frac{11}{3}\pi\right) =$

$= 0 - 1 - 2\cos\left(\frac{8\pi}{4} - \frac{\pi}{4}\right) + \operatorname{sen}\left(\frac{11\pi}{3}\right)$

$= -1 - 2\cos\left(-\frac{\pi}{4}\right) + \operatorname{sen}\left(\frac{12\pi}{3} - \frac{\pi}{3}\right)$

$= -1 - 2\cos\left(\frac{\pi}{4}\right) + \operatorname{sen}\left(-\frac{\pi}{3}\right) =$

$= -1 - \cancel{2}\frac{\sqrt{2}}{\cancel{2}} - \operatorname{sen}\frac{\pi}{3} = -1 - \sqrt{2} - \frac{\sqrt{3}}{2}$

$= \frac{-2 - 2\sqrt{2} - \sqrt{3}}{2} = -\frac{2 + 2\sqrt{2} + \sqrt{3}}{2}$

1.9 $-5\operatorname{sen}\left(\frac{43}{6}\pi\right) + 7\cos\left(-\frac{11\pi}{6}\right) + 3\operatorname{sen}\left(\frac{27\pi}{4}\right) =$

$= -5\operatorname{sen}\left(\frac{42}{6}\pi + \frac{\pi}{6}\right) + 7\cos\left(-\frac{12\pi}{6} + \frac{\pi}{6}\right) + 3\operatorname{sen}\left(\frac{28\pi}{4} - \frac{\pi}{4}\right)$

$= -5\operatorname{sen}\left(7\pi + \frac{\pi}{6}\right) + 7\cos\left(-2\pi + \frac{\pi}{6}\right) + 3\operatorname{sen}\left(7\pi - \frac{\pi}{4}\right) =$

$= 5\operatorname{sen}\frac{\pi}{6} + 7\cos\frac{\pi}{6} + 3\operatorname{sen}\frac{\pi}{4} =$

$= 5 \times \frac{1}{2} + 7 \times \frac{\sqrt{3}}{2} + 3\frac{\sqrt{2}}{2} = \frac{5 + 7\sqrt{3} + 3\sqrt{2}}{2}$

1.10 $$\frac{\operatorname{sen}\left(-\frac{7}{6}\pi\right)+\cos^2\left(-\frac{13}{4}\pi\right)}{\tan\left(\frac{8}{3}\pi\right)}=$$

$$=\frac{\operatorname{sen}\left(-\frac{6}{6}\pi-\frac{\pi}{6}\right)+\cos^2\left(-\frac{12}{4}\pi-\frac{\pi}{4}\right)}{\tan\left(\frac{9}{3}\pi-\frac{\pi}{3}\right)}=$$

$$=\frac{\operatorname{sen}\frac{\pi}{6}-\cos^2\frac{\pi}{4}}{-\tan\frac{\pi}{3}}=\frac{\frac{1}{2}-\left(\frac{\sqrt{2}}{2}\right)^2}{-\sqrt{3}}=$$

$$=\frac{\frac{1}{2}-\frac{2}{4}}{-\sqrt{3}}=0$$

2.1 $$\operatorname{sen}(\pi-\alpha)+\cos\left(\frac{\pi}{2}-\alpha\right)+2\operatorname{sen}(-\alpha)=$$

$$=\operatorname{sen}\alpha+\operatorname{sen}\alpha-2\operatorname{sen}\alpha=0$$

2.2 $$\tan(-\alpha)\times\operatorname{sen}\left(\frac{\pi}{2}+\alpha\right)+\operatorname{sen}(\pi+\alpha)=$$

$$=-\tan\alpha\times\cos\alpha-\operatorname{sen}\alpha=$$

$$=-\frac{\operatorname{sen}\alpha}{\cancel{\cos\alpha}}\times\cancel{\cos\alpha}-\operatorname{sen}\alpha=-2\operatorname{sen}\alpha$$

2.3 $\operatorname{sen}^2\left(\frac{\pi}{2}+\alpha\right)+\operatorname{sen}^2(5\pi-\alpha)=$

$=\cos^2\alpha+\operatorname{sen}^2\alpha=1$

3. $\cos(\pi-\alpha)-2\operatorname{sen}(\alpha-\pi)=$

$=-\cos\alpha-2\operatorname{sen}(-(\pi-\alpha))=$

$=-\cos\alpha+2\operatorname{sen}(\pi-\alpha)=$

$=-\cos\alpha+2\operatorname{sen}\alpha$

4.1 $a\cos\left(-\frac{3\pi}{4}\right)=\tan\left(-\frac{7\pi}{4}\right)+\operatorname{sen}\left(-\frac{11\pi}{6}\right)\Leftrightarrow$

$\Leftrightarrow a\cos\left(-\pi+\frac{\pi}{4}\right)=\tan\left(-\frac{8\pi}{4}+\frac{\pi}{4}\right)+\operatorname{sen}\left(-\frac{12}{6}\pi+\frac{\pi}{6}\right)$

$\Leftrightarrow -a\cos\left(\frac{\pi}{4}\right)=\tan\left(\frac{\pi}{4}\right)+\operatorname{sen}\left(\frac{\pi}{6}\right)=$

$\Leftrightarrow -a\frac{\sqrt{2}}{2}=1+\frac{1}{2}\Leftrightarrow -a\frac{\sqrt{2}}{2}=\frac{3}{2}\Leftrightarrow$

$\Leftrightarrow \sqrt{2}\,a=-3\Leftrightarrow a=-\frac{3}{\sqrt{2}}\Leftrightarrow a=-\frac{3\sqrt{2}}{2}$

4.2

$$a \operatorname{sen}\left(-\frac{2\pi}{3}\right) + a\cos(-\pi) = \tan\left(-\frac{3\pi}{4}\right) + \cos\left(\frac{2\pi}{3}\right)$$

$$a \operatorname{sen}\left(-\frac{3\pi}{3} + \frac{\pi}{3}\right) + a \times (-1) = \tan\left(-\frac{4\pi}{4} + \frac{\pi}{4}\right) + \cos\left(\frac{3\pi}{3} - \frac{\pi}{3}\right)$$

$$-a \operatorname{sen}\frac{\pi}{3} - a = \tan\frac{\pi}{4} - \cos\frac{\pi}{3}$$

$$-a\frac{\sqrt{3}}{2} - a = 1 - \frac{1}{2}$$

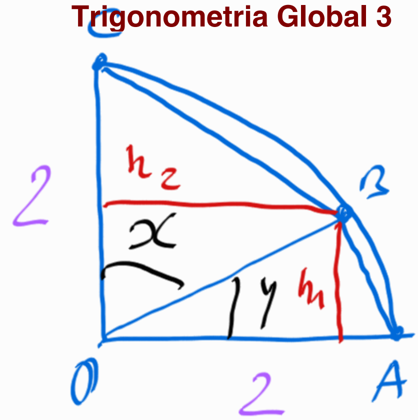
$$-a\left(\frac{\sqrt{3}}{2} + 1\right) = \frac{1}{2}$$

$$a = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2} + 1} \Leftrightarrow a = \frac{-\frac{1}{2}\left(\frac{\sqrt{3}}{2} - 1\right)}{\left(\frac{\sqrt{3}}{2} + 1\right)\left(\frac{\sqrt{3}}{2} - 1\right)}$$

$$a = \frac{-\frac{\sqrt{3}}{4} + \frac{1}{2}}{\frac{3}{4} - 1} \Leftrightarrow a = \frac{\frac{-\sqrt{3} + 2}{\cancel{4}}}{-\frac{1}{\cancel{4}}}$$

$$a = \sqrt{3} - 2$$

5. $y = \frac{\pi}{2} - x$



$$A_{[OABC]} = A_{[OAB]} + A_{[OBC]}$$

$$A_{[OAB]} = \frac{\overline{OA} \times h_1}{2} = \frac{2 \times h_1}{2} = h_1$$

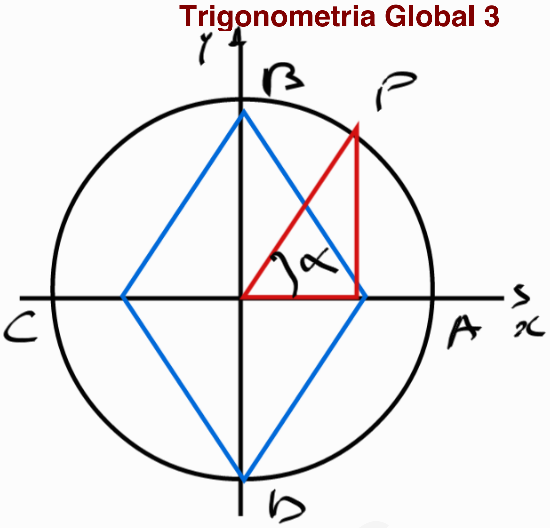
$$h_1 = 2 \operatorname{sen} y = 2\left(\operatorname{sen}\left(\frac{\pi}{2} - x\right)\right)$$
$$= 2 \cos x$$

$$A_{[OBC]} = \frac{\overline{OC} \times h_2}{2} = \frac{2 \times h_2}{2} = h_2$$

$$h_2 = 2 \cos y = 2 \cos\left(\frac{\pi}{2} - x\right) =$$
$$= 2 \operatorname{sen} x$$

$$A_{[OABC]} = 2 \cos x + 2 \operatorname{sen} x$$
$$= 2(\operatorname{sen} x + \cos x)$$

A



6. $[ABCD]$ é um losango logo

$\overline{AB} = \overline{BC} = \overline{CD} = \overline{AD}$

$\overline{OP} = \overline{OB} = 1$ são raios da circunferência.

$\overline{OA} = \cos\alpha$

$\overline{AB}^2 = \overline{OA}^2 + \overline{OB}^2 \Leftrightarrow \overline{AB}^2 = \cos^2\alpha + 1$

$\Leftrightarrow \overline{AB} = \sqrt{\cos^2\alpha + 1}\ ,\ \overline{AB} > 0$

$P_{[ABCD]} = 4 \times \sqrt{\cos^2\alpha + 1} = 4\sqrt{\cos^2\alpha + 1}$ C

7. $\operatorname{sen}\left(\alpha - \frac{\pi}{2}\right) = \operatorname{sen}\left(-\left(\frac{\pi}{2} - \alpha\right)\right) =$

$= -\operatorname{sen}\left(\frac{\pi}{2} - \alpha\right) = -\cos\alpha = \cos(\pi - \alpha)$ C

8. $d(t) = \cos(\pi t) + 12$

8.1 $d(t) = 11{,}5 \Leftrightarrow \cos(\pi t) + 12 = 11{,}5$

$\Leftrightarrow \cos(\pi t) = -0{,}5 \Leftrightarrow \cos(\pi t) = -\frac{1}{2} \Leftrightarrow$

$\Leftrightarrow \pi t = -\frac{\pi}{3} + 2k\pi \vee \pi t = \frac{\pi}{3} + 2k\pi\ ,\ k \in \mathbb{Z}$

$\Leftrightarrow t = -\frac{1}{3} + 2k \vee t = \frac{1}{3} + 2k\ ,\ k \in \mathbb{Z}$

Como $t \geq 0$ então $t = \frac{1+6k}{3}\ ,\ k \in \mathbb{Z}^+$

8.2 $-1 \le \cos(\pi t) \le 1 \Leftrightarrow$

$$11 \le 12+\cos(\pi t) \le 13$$

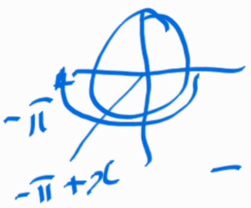
8.3 $d(t+P) = d(t) \Leftrightarrow$

$$\Leftrightarrow 12+\cos(\pi(t+P)) = 12+\cos(\pi t)$$

$$\Leftrightarrow 12+\cos(\pi t+\pi P) = 12+\cos(\pi t)$$

$$\pi P = 2\pi \Leftrightarrow P = 2$$

9. $\cos(x-\pi)\tan x + \operatorname{sen}\left(2x-\frac{3}{2}\pi\right)\cos 2x - \operatorname{sen}^2 2x - \operatorname{sen}(-x)$



$-\cos x$

$\cos(2x)$

$-\operatorname{sen}(-x)$
$= \operatorname{sen} x$

$$= -\cancel{\cos x}\cdot\frac{\cancel{\operatorname{sen} x}}{\cancel{\cos x}} + \cos 2x \cos 2x - \operatorname{sen}^2 2x + \cancel{\operatorname{sen} x}$$

$$= \cos^2(2x) - \operatorname{sen}^2(2x) = 1 - \operatorname{sen}^2(2x) - \operatorname{sen}^2(2x)$$

$$= 1 - 2\operatorname{sen}^2(2x) \quad \text{c.q.m.}$$

g é par: $g(-x) = 1 - 2\operatorname{sen}^2(-2x) =$

$$= 1 - 2\left(-\operatorname{sen}(2x)\right)^2 = 1 - 2\operatorname{sen}^2(2x)$$

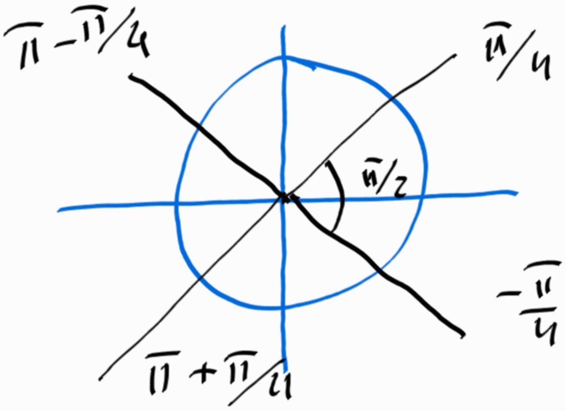
$$= g(x) \quad \text{logo } g \text{ é par.}$$

Zeros: $1 - 2\operatorname{sen}^2(2x) = 0 \Leftrightarrow \operatorname{sen}^2(2x) = \frac{1}{2}$

$\Leftrightarrow \operatorname{sen}(2x) = \pm\sqrt{\frac{1}{2}} \Leftrightarrow \operatorname{sen}(2x) = \pm\frac{\sqrt{2}}{2}$

$\Leftrightarrow 2x = \frac{\pi}{4} + 2k\pi \vee 2x = \pi - \frac{\pi}{4} + 2k\pi \vee$

$\vee\ 2x = -\frac{\pi}{4} + 2k\pi \vee 2x = \pi + \frac{\pi}{4} + 2k\pi, k \in \mathbb{Z}$



logo $2x = \frac{\pi}{4} + \frac{2k\pi}{2} \Leftrightarrow x = \frac{\pi}{8} + \frac{k\pi}{2}, k \in \mathbb{Z}$

10. $\cos(3x) = 2\cos^2(3x) \Leftrightarrow \cos(3x) - 2\cos^2(3x) = 0$

$\Leftrightarrow \cos(3x)\left(1 - 2\cos(3x)\right) = 0$

$\Leftrightarrow \cos(3x) = 0 \vee 1 - 2\cos(3x) = 0$

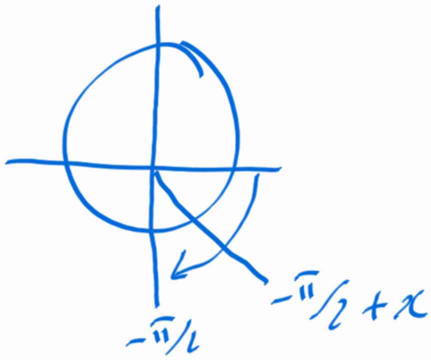
$\Leftrightarrow \cos(3x) = 0 \vee \cos(3x) = \frac{1}{2}$

$\Leftrightarrow 3x = \frac{\pi}{2} + k\pi \vee 3x = \frac{\pi}{3} + 2k\pi \vee 3x = -\frac{\pi}{3} + 2k\pi,$
$k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{\pi}{6} + \frac{k\pi}{3} \vee x = \frac{\pi}{9} + \frac{2k\pi}{3} \vee x = -\frac{\pi}{9} + \frac{2k\pi}{3},$
$k \in \mathbb{Z}$

11. Não é para fazer

12. $f(x) = \cos\left(x - \frac{\pi}{2}\right)$

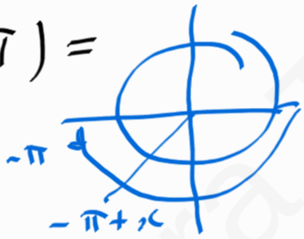


$= \operatorname{sen} x$

A

$f(x) = \operatorname{sen} x$

$f(x - \pi) = \operatorname{sen}(x - \pi) =$



$= -\operatorname{sen} x$

Falso

B

f é par.

A função seno é ímpar.

Falso

C

$f(x) = \operatorname{sen} x$

$-f(-x) = -\operatorname{sen}(-x) = \operatorname{sen} x$

Verdade

C

13. $\alpha \in]0, \frac{\pi}{4}[$; $\alpha + \beta = \pi \Leftrightarrow \beta = \pi - \alpha$

$\alpha + \theta = \frac{3\pi}{2} \Leftrightarrow \theta = \frac{3\pi}{2} - \alpha$

$\operatorname{sen}\alpha = \operatorname{sen}\alpha$ / $\cos\beta = \cos(\pi - \alpha) = -\cos\alpha$

$\operatorname{sen}\theta = \operatorname{sen}\left(\frac{3\pi}{2} - \alpha\right) = -\cos\alpha$

$\operatorname{sen}\alpha + \cos\beta + \operatorname{sen}\theta = \operatorname{sen}\alpha - 2\cos\alpha$

D

14. $5+2\cos x=6 \Leftrightarrow \cos x=\frac{1}{2}$

$\Leftrightarrow x=\frac{\pi}{3}+2k\pi \vee x=-\frac{\pi}{3}+2k\pi \ , k\in\mathbb{Z}$

$[0, 2\pi]$

$k=0 \Rightarrow x=\frac{\pi}{3} \vee x=-\frac{\pi}{3}$

$k=1 \Rightarrow x=\frac{\pi}{3}+2\pi \vee x=-\frac{\pi}{3}+2\pi$

$x=\frac{5\pi}{3}$

$C.S=\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$ A

15. $1+\sqrt{3}\tan(3x+\pi)=2 \Leftrightarrow \tan(3x+\pi)=\frac{1}{\sqrt{3}} \Leftrightarrow$

$\Leftrightarrow \tan(3x+\pi)=\frac{\sqrt{3}}{3}$

$\Leftrightarrow 3x+\pi=\frac{\pi}{6}+k\pi \ , k\in\mathbb{Z}$

$\Leftrightarrow 3x=\frac{\pi}{6}-\pi+k\pi \ , k\in\mathbb{Z}$

$\Leftrightarrow 3x=-\frac{5\pi}{6}+k\pi \ , k\in\mathbb{Z}$

$\Leftrightarrow x=-\frac{5\pi}{18}+\frac{k\pi}{3} \ , k\in\mathbb{Z}$

$k=-2 \Rightarrow x=-\frac{5\pi}{18}-\frac{2\pi}{3}=-\frac{17\pi}{18} \notin [-2\pi, -\pi]$

$k=-3 \Rightarrow x=-\frac{5\pi}{18}-\pi=-\frac{23\pi}{18} \in [-2\pi, -\pi]$

$k=-4 \Rightarrow x=-\frac{5\pi}{18}-\frac{4\pi}{3}=-\frac{29\pi}{18} \in [-2\pi, -\pi]$

$k=-5 \Rightarrow x=-\frac{5\pi}{18}-\frac{5\pi}{3}=-\frac{35\pi}{18} \in [-2\pi, -\pi]$ C

16. A $\frac{1}{\operatorname{sen}^2 x}$ $D = \{x \in \mathbb{R} : \operatorname{sen}^2 x \neq 0\}$

$\operatorname{sen}^2 x = 0 \Leftrightarrow \operatorname{sen} x = 0 \Leftrightarrow$

$x = k\pi , k \in \mathbb{Z}$

B $\frac{1}{\tan^2 x + 1}$ $D = \{x \in \mathbb{R} : \tan^2 x + 1 \neq 0\}$

$\tan^2 x + 1 = 0 \Leftrightarrow \frac{1}{\cos^2 x} = 0$

$\Leftrightarrow \cos^2 x = 0 \Leftrightarrow \cos x = 0$

$\Leftrightarrow x = \frac{\pi}{2} + k\pi , k \in \mathbb{Z}$

C $\frac{1}{1 + \operatorname{sen} x}$ $D = \{x \in \mathbb{R} : 1 + \operatorname{sen} x \neq 0\}$

$1 + \operatorname{sen} x = 0 \Leftrightarrow \operatorname{sen} x = -1$

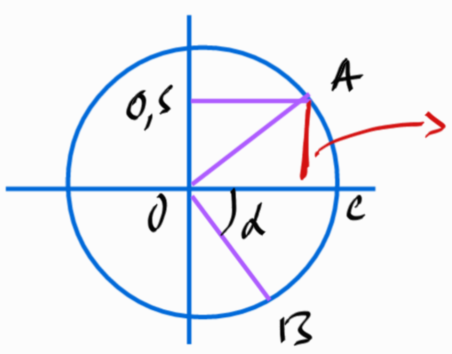
$x = \frac{3\pi}{2} + 2k\pi , k \in \mathbb{Z}$

D $\sqrt{\cos x + 1}$ $D = \{x \in \mathbb{R} : \cos x + 1 \geq 0\} = \mathbb{R}$

$\cos x + 1 \geq 0 \Leftrightarrow \cos x \geq -1$

D

17.



$A\left(0{,}5=\cos\frac{\pi}{3},\ \operatorname{sen}\frac{\pi}{3}=\frac{\sqrt{3}}{2}\right)$

$A\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$

$\frac{\pi}{2}-\frac{\pi}{3}=\frac{\pi}{6}$

então $\alpha=-\frac{\pi}{6}$

$\tan(\pi-\alpha)+\cos\alpha=$

$\tan\left(\pi+\frac{\pi}{6}\right)+\cos\left(-\frac{\pi}{6}\right)=-\tan\frac{\pi}{6}+\cos\frac{\pi}{6}=$

$=-\frac{\sqrt{3}}{3}+\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{6}$

D

18. $\operatorname{sen}x+\cos x=4$

A

Recorrendo à calculadora gráfica

19.1

$$A_{[BCDQP]} = A_{[BCDOP]} + A_{[OPQ]}$$

$$A_{[BCDOP]} = A_{[ABCD]} - A_{[AOP]}$$

$$A_{[ABCD]} = 2 \times 1 = 2$$

$$A_{[AOP]} = \frac{\overline{AP} \times \overline{AO}}{2} = \qquad \overline{AP} = \tan\alpha$$

$$\overline{AO} = 1$$

$$= \frac{\tan\alpha}{2}$$

$$A_{[BCDOP]} = 2 - \frac{\tan\alpha}{2}$$

$$A_{[ODQ]} = \frac{\overline{OD} \times \overline{QR}}{2} = \frac{1 \times \operatorname{sen}\alpha}{2}$$

$$= \frac{\operatorname{sen}\alpha}{2}$$

$$A_{[BCDQP]} = 2 - \frac{\tan\alpha}{2} + \frac{\operatorname{sen}\alpha}{2}$$

c.q.m.

19.2 $\cos\left(\frac{3\pi}{2}-x\right) = -\frac{3}{5} \Leftrightarrow$

$\Leftrightarrow -\text{sen}\, x = -\frac{3}{5}$

$\Leftrightarrow \text{sen}\, x = \frac{3}{5}$

$\text{sen}^2 x + \cos^2 x = 1 \Leftrightarrow \left(\frac{3}{5}\right)^2 + \cos^2 x = 1$

$\Leftrightarrow \frac{9}{25} + \cos^2 x = 1 \Leftrightarrow \cos^2 x = 1 - \frac{9}{25}$

$\Leftrightarrow \cos x = \pm\sqrt{\frac{16}{25}}$, $x \in \left]0, \frac{\pi}{4}\right[$

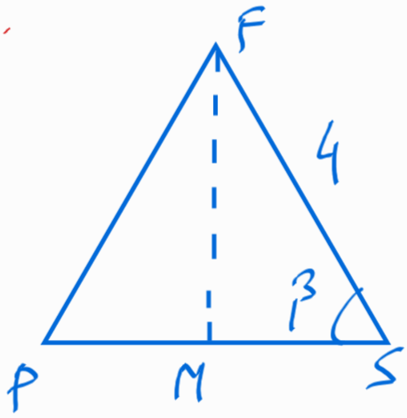
$\Leftrightarrow \cos x = \frac{4}{5}$

$\tan x = \frac{\text{sen}\, x}{\cos x} = \frac{\frac{3}{\cancel{5}}}{\frac{4}{\cancel{5}}} = \frac{3}{4}$

$A = 2 - \frac{\frac{3}{4}}{2} + \frac{\frac{3}{5}}{2} = 2 - \frac{3}{8} + \frac{3}{10}$

$= \frac{77}{40}$

20.



β é a amplitude de $F\hat{S}P$

M é o ponto médio de [PS]

$$A_{[FPS]} = \frac{\overline{PS} \times \overline{FM}}{2} = \frac{\cancel{2} \times \overline{MS} \times \overline{FM}}{\cancel{2}}$$

$$= \overline{MS} \times \overline{FM}$$

$\overline{MS} = 4 \operatorname{sen} \beta$ e $\overline{FM} = 4 \cos \beta$

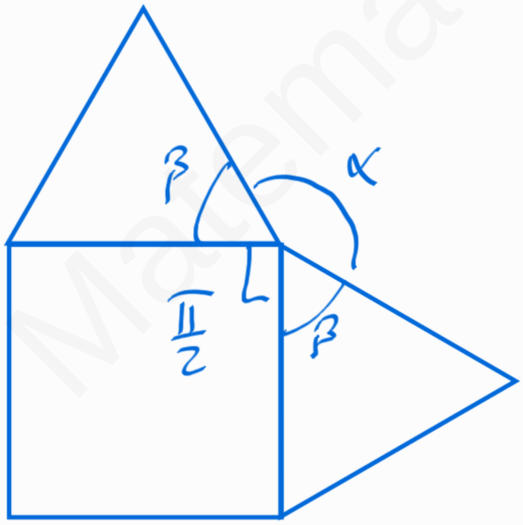
Assim, $A_{[FPS]} = 4 \operatorname{sen} \beta \times 4 \cos \beta = 16 \operatorname{sen} \beta \cos \beta$

Atendendo à sugestão do enunciado

$\operatorname{sen}(2\beta) = 2 \operatorname{sen} \beta \cos \beta$

Logo $16 \operatorname{sen} \beta \cos \beta = 8 \times 2 \operatorname{sen} \beta \cos \beta$

$= 8 \operatorname{sen}(2\beta)$



Observando a figura temos:

$$2\beta + \alpha + \frac{\pi}{2} = 2\pi \Leftrightarrow$$

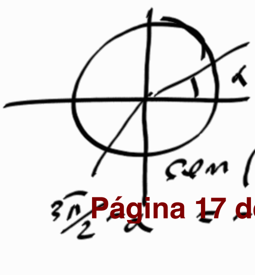
$$\Leftrightarrow 2\beta = 2\pi - \frac{\pi}{2} - \alpha$$

$$\Leftrightarrow 2\beta = \frac{3\pi}{2} - \alpha$$

Substituindo: $8 \operatorname{sen}(2\beta) = 8 \operatorname{sen}\left(\frac{3\pi}{2} - \alpha\right) =$

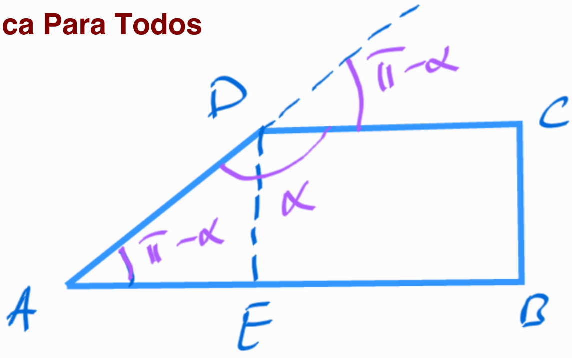
$= -8 \cos \alpha$

Portanto a área lateral da pirâmide é $4 \times (-8 \cos \alpha) = -32 \cos \alpha$ u. a.



$\operatorname{sen}\left(\frac{3\pi}{2} - \alpha\right) = -\cos \alpha$

21.1



$\overline{BC} = 1 = \overline{DE}$

$\overline{CD} = 1$

$$P_{[ABCD]} = \overline{AB} + \overline{BC} + \overline{CD} + \overline{DA}$$

$$\overline{AB} = \overline{AE} + \underbrace{\overline{EB}}_{=1}$$

$\overline{EB} + \overline{BC} + \overline{CD} = 3$

$$P_{[ABCD]} = 3 + \overline{AE} + \overline{DA}$$

$$\operatorname{sen}(\pi - \alpha) = \frac{\overline{ED}}{\overline{AD}} \Leftrightarrow \overline{AD} = \frac{1}{\operatorname{sen}(\pi-\alpha)}$$

$\operatorname{sen}(\pi - \alpha) = \operatorname{sen}\alpha$

$$\overline{AD} = \frac{1}{\operatorname{sen}\alpha}$$

$$\tan(\pi - \alpha) = \frac{\overline{ED}}{\overline{AE}} \Leftrightarrow \overline{AE} = \frac{1}{\tan(\pi-\alpha)}$$

$\tan(\pi - \alpha) = -\tan\alpha$

$$\overline{AE} = -\frac{1}{\tan\alpha}$$

Assim $P(\alpha) = 3 - \dfrac{1}{\tan\alpha} + \dfrac{1}{\operatorname{sen}\alpha}$

$$= 3 - \frac{\cos\alpha}{\operatorname{sen}\alpha} + \frac{1}{\operatorname{sen}\alpha}$$

$$= 3 + \frac{1 - \cos\alpha}{\operatorname{sen}\alpha}$$ c.q.m.

21.2 $\tan\theta = -\sqrt{8}$, $\theta \in \left]\frac{\pi}{2}, \pi\right[$

$$\tan^2\theta + 1 = \frac{1}{\cos^2\theta} \Leftrightarrow (-\sqrt{8})^2 + 1 = \frac{1}{\cos^2\theta} \Leftrightarrow$$

$$\Leftrightarrow 8 + 1 = \frac{1}{\cos^2\theta} \Leftrightarrow 9 = \frac{1}{\cos^2\theta} \Leftrightarrow \cos^2\theta = \frac{1}{9}$$

$$\Leftrightarrow \cos\theta = \pm\sqrt{\frac{1}{9}} \Leftrightarrow \cos\theta = -\frac{1}{3}$$

$\theta \in 2^{o}Q.$
$\cos\theta < 0$

$$\operatorname{sen}^2\theta + \cos^2\theta = 1 \Leftrightarrow \operatorname{sen}^2\theta = 1 - \cos^2\theta$$

$$\Leftrightarrow \operatorname{sen}^2\theta = 1 - \left(-\frac{1}{3}\right)^2 \Leftrightarrow \operatorname{sen}^2\theta = 1 - \frac{1}{9}$$

$$\Leftrightarrow \operatorname{sen}^2\theta = \frac{8}{9} \Leftrightarrow \operatorname{sen}\theta = \pm\sqrt{\frac{8}{9}} \Leftrightarrow$$

$$\Leftrightarrow \operatorname{sen}\theta = \frac{2\sqrt{2}}{3}$$

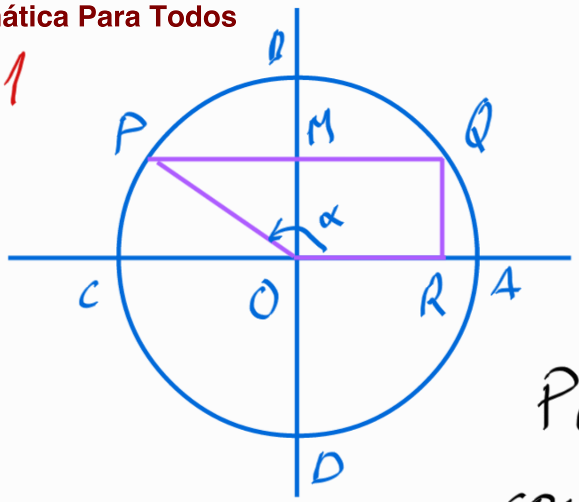
$\theta \in 2^{o}Q.$
$\operatorname{sen}\theta > 0$

Portanto $f(\theta) = 3 + \dfrac{1 - \left(-\frac{1}{3}\right)}{\frac{2\sqrt{2}}{3}} =$

$$= 3 + \frac{4/\cancel{3}}{2\sqrt{2}/\cancel{3}} = 3 + \frac{4}{2\sqrt{2}} =$$

$$= 3 + \frac{2}{\sqrt{2}} = 3 + \sqrt{2}$$

22.1



$$A_{[PQRO]} = \frac{\overline{PQ} + \overline{OR}}{2} \times \overline{QR}$$

$P(\cos\alpha, \operatorname{sen}\alpha)$

como P e Q têm a mesma ordenada então $\overline{PM} = \overline{MQ} = \overline{OR}$

assim $\overline{PQ} = 2 \times \overline{OR}$

como $\alpha \in \left]\frac{\pi}{2}, \pi\right[$

$\overline{OR} = -\cos\alpha$, porque $\cos\alpha < 0$

$\overline{PQ} = -2\cos\alpha$

$\overline{QR} = \operatorname{sen}\alpha$

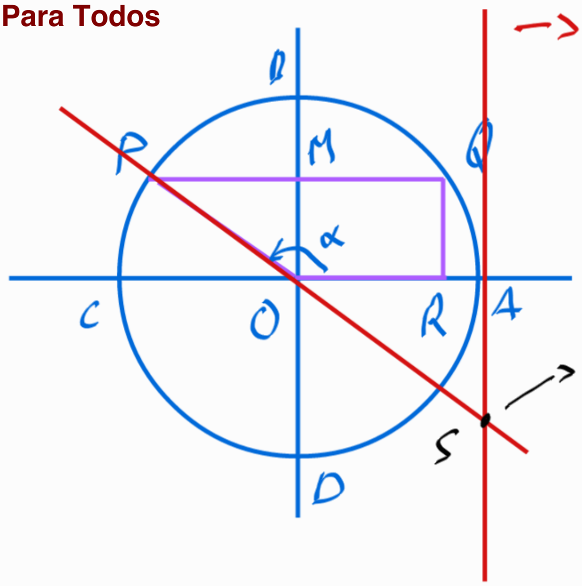
Portanto

$$A_{[PQRO]} = \frac{-\cos\alpha + (-\cos\alpha)}{2} \times \operatorname{sen}\alpha$$

$$= -\frac{3\cos\alpha}{2} \times \operatorname{sen}\alpha$$

$$= -\frac{3}{2}\cos\alpha \operatorname{sen}\alpha \quad \text{c.q.m.}$$

22.2



$\rightarrow S\left(1,-\frac{7}{24}\right)$, isto é,

$\tan\alpha = -\frac{7}{24}$

Assim $1+\tan^2\alpha = \frac{1}{\cos^2\alpha} \Leftrightarrow 1+\left(-\frac{7}{24}\right)^2 = \frac{1}{\cos^2\alpha} \Leftrightarrow$

$\Leftrightarrow 1+\frac{49}{576} = \frac{1}{\cos^2\alpha} \Leftrightarrow \frac{625}{576} = \frac{1}{\cos^2\alpha} \Leftrightarrow \cos^2\alpha = \frac{576}{625}$

$\Leftrightarrow \cos\alpha = \pm\sqrt{\frac{576}{625}} \Leftrightarrow \cos\alpha = -\frac{24}{25}$

$\alpha \in \left]\frac{\pi}{2},\pi\right[$

$\cos\alpha < 0$

$\tan\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha} \Leftrightarrow \operatorname{sen}\alpha = \tan\alpha\cos\alpha$

$\Leftrightarrow \operatorname{sen}\alpha = \left(-\frac{7}{\cancel{24}}\right)\times\left(-\frac{\cancel{24}}{25}\right)$

$\Leftrightarrow \operatorname{sen}\alpha = \frac{7}{25}$

Portanto

$A = -\frac{3}{2}\operatorname{sen}\alpha\cos\alpha = -\frac{3}{2}\left(\frac{7}{25}\right)\times\left(-\frac{24}{25}\right)$

$= \frac{252}{625}$

23. Qualquer triângulo inscrito numa semicircunferência é retângulo.

$$A_{\text{região sombreada}} = A_{1\,(\text{semicircunferência})} - A_{2\,(\text{triângulo})}$$

$$A_1 = \frac{\pi \times r^2}{2} = \frac{2^2 \pi}{2} = \frac{4\pi}{2} = 2\pi$$

$$A_2 = \frac{\overline{AB} \times \overline{BC}}{2}$$

$$\cos\alpha = \frac{\overline{AB}}{4} \Leftrightarrow \overline{AB} = 4\cos\alpha$$

$$\operatorname{sen}\alpha = \frac{\overline{BC}}{4} \Leftrightarrow \overline{BC} = 4\operatorname{sen}\alpha$$

$$A_2 = \frac{4\cos\alpha \times 4\operatorname{sen}\alpha}{2} = 8\cos\alpha\operatorname{sen}\alpha$$

logo $A_{\text{sombreada}} = 2\pi - 8\cos\alpha\operatorname{sen}\alpha$ c.q.m.

24. Seja P a projeção ortogonal do ponto C na reta AB, assim [CP] é a altura do triângulo [ABC], relativamente à base [AB].

$$A_{[ABC]} = \frac{\overline{AB} \times \overline{CP}}{2}$$

$\overline{AB} = \tan\alpha$, logo

$$\tan^2\alpha + 1 = \frac{1}{\cos^2\alpha} \Leftrightarrow \overline{AB}^2 + 1 = \frac{1}{\cos^2\alpha} \Leftrightarrow \overline{AB}^2 + 1 = \frac{1}{\left(\frac{1}{3}\right)^2} \Leftrightarrow$$

$$\Leftrightarrow \overline{AB}^2 = \frac{1}{\frac{1}{9}} - 1 \Leftrightarrow \overline{AB}^2 = 9 - 1 \Leftrightarrow \overline{AB} = \pm\sqrt{8} \Leftrightarrow$$

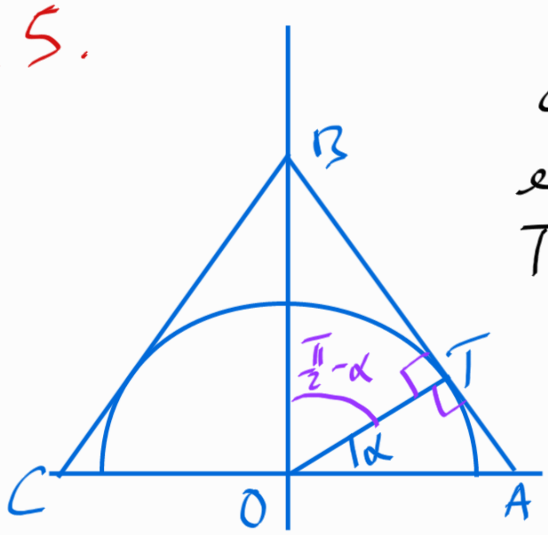
$$\Leftrightarrow \overline{AB} = 2\sqrt{2}$$

$\alpha \in 1^o Q.$
$\cos\alpha > 0$

$$\overline{CP} = 1 - \underbrace{x_C}_{C(\cos\alpha,\ \operatorname{sen}\alpha)} = 1 - \cos\alpha = 1 - \frac{1}{3} = \frac{2}{3}$$

$$A_{[ABC]} = \frac{2\sqrt{2} \times \frac{2}{3}}{2} = \frac{2\sqrt{2}}{3}$$

25.



Como AB é tangente à semicircunferência no ponto T e como [OT] é um raio, então $AB \perp OT$ logo [OTA] e [OTB] são triângulos retângulos

$$A_{[ABC]} = \frac{\overline{AC} \times \overline{OB}}{2} = \frac{\cancel{2} \times \overline{OA} \times \overline{OB}}{\cancel{2}} = \overline{OA} \times \overline{OB}$$

$$\cos\alpha = \frac{\overline{OT}}{\overline{OA}} \Leftrightarrow \overline{OA} = \frac{2}{\cos\alpha}$$

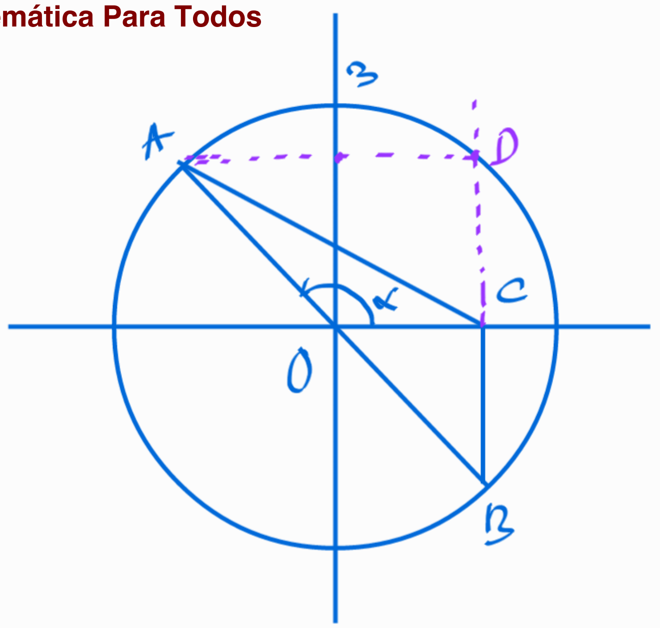
$$\cos\left(\frac{\pi}{2}-\alpha\right) = \frac{\overline{OT}}{\overline{OB}} \Leftrightarrow \operatorname{sen}\alpha = \frac{2}{\overline{OB}} \Leftrightarrow$$

$$\Leftrightarrow \overline{OB} = \frac{2}{\operatorname{sen}\alpha}$$

Portanto $A_{[ABC]} = \frac{2}{\cos\alpha} \times \frac{2}{\operatorname{sen}\alpha} = \frac{4}{\operatorname{sen}\alpha\cos\alpha}$

c.q.m.

26.



Seja D a projeção ortogonal do ponto A na reta BC, então [AD] é a altura do triângulo [ABC], relativamente à base [BC]

$$A_{[ABC]} = \frac{\overline{AD} \times \overline{BC}}{2}$$

O ponto A é simétrico do ponto B em relação à origem.

As coordenadas de A são $(3\cos\alpha, 3\,\text{sen}\,\alpha)$

logo as coordenadas de B são $(-3\cos\alpha, -3\,\text{sen}\,\alpha)$

$\overline{BC} = |-3\,\text{sen}\,\alpha| = 3\,\text{sen}\,\alpha$

$\overline{AD} = 2 \times \overline{OC}$

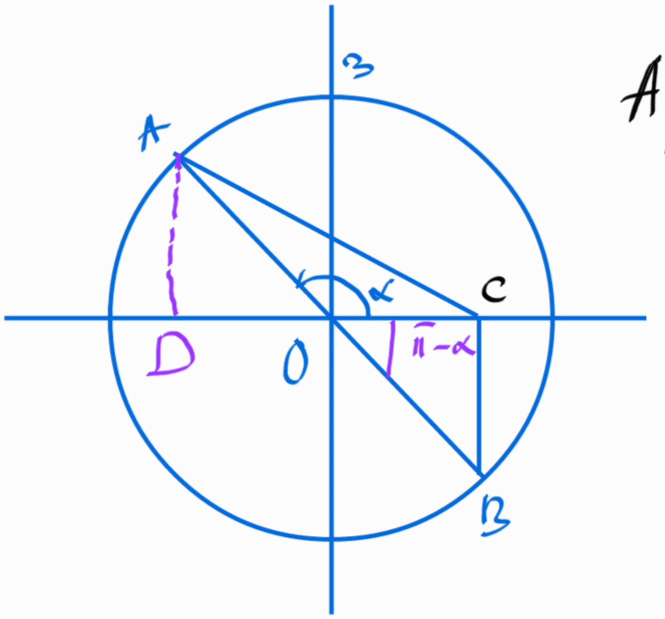
$C(-3\cos\alpha, 0)$ C tem a mesma abcissa de B.

$\overline{OC} = -3\cos\alpha$, $\cos\alpha < 0$,

$\overline{AD} = -6\cos\alpha$

$$A_{[ABC]} = \frac{-6\cos\alpha \times 3\,\text{sen}\,\alpha}{2} = -9\cos\alpha\,\text{sen}\,\alpha$$ c.q.m.

Outro processo



$$A_{[ABC]} = A_{[AOC]} + A_{[OBC]}$$

$A(3\cos\alpha, 3\operatorname{sen}\alpha)$

$B(3\cos(\pi-\alpha), 3\operatorname{sen}(\pi-\alpha))$

$B(\underbrace{-3\cos\alpha}_{\substack{\cos\alpha<0 \\ e \\ B\in 4^{o}Q}}, \underbrace{-3\operatorname{sen}\alpha}_{\substack{\operatorname{sen}\alpha>0 \\ e \\ B\in 4^{o}Q}})$

$C(\underbrace{-3\cos\alpha}_{\cos\alpha<0}, 0)$

e em C o cosseno é positivo

$$A_{[AOC]} = \frac{\overline{AD} \times \overline{OC}}{2}$$

$\overline{AD} = 3\operatorname{sen}\alpha$
$\operatorname{sen}\alpha > 0$

$\overline{OC} = -3\cos\alpha$
$\cos\alpha < 0$

$$= \frac{3\operatorname{sen}\alpha \times (-3\cos\alpha)}{2} = -\frac{9\operatorname{sen}\alpha\cos\alpha}{2}$$

$$A_{[OBC]} = \frac{\overline{OC} \times \overline{BC}}{2}$$

$\overline{OC} = -3\cos\alpha$
$\cos\alpha < 0$

$\overline{BC} = |-3\operatorname{sen}\alpha| =$
$= 3\operatorname{sen}\alpha$
$\operatorname{sen}\alpha > 0$

$$= \frac{-3\cos\alpha \cdot 3\operatorname{sen}\alpha}{2}$$

$$= -\frac{9\cos\alpha\operatorname{sen}\alpha}{2}$$

$$A_{[ABC]} = -\frac{9\cos\alpha\operatorname{sen}\alpha}{2} + \frac{-9\cos\alpha\operatorname{sen}\alpha}{2} = -\frac{9\operatorname{sen}\alpha\cos\alpha}{2}$$

c.q.m.