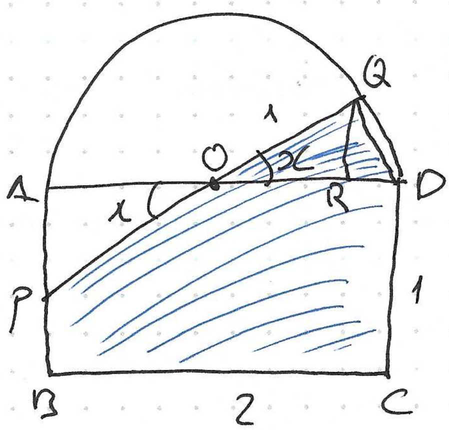
TRIGONOMETRIA

EXERCÍCIOS DE EXAME / TESTES INTERMÉDIOS 1

1.1 A área do polígono [BCDQP] é igual à área do triângulo [ODQ] com a área do pentágono [ODCBP]

$$A_{[ODQ]} = \frac{\overline{OD} \times \overline{QR}}{2}$$



$$\text{sen}\, x = \frac{\overline{QR}}{1} \Leftrightarrow \overline{QR} = \text{sen}\, x$$

$$\overline{OD} = 1$$

$$A_{[ODQ]} = \frac{1 \times \text{sen}\, x}{2}$$

$$= \frac{\text{sen}\, x}{2}$$

Área do pentágono [ODCBP] é igual à diferença entre a área do retângulo [ABCD] e a área do triângulo [OAP]

$$A_{[ODCBP]} = A_{[ABCD]} - A_{[OAP]}$$

$$A_{[OAP]} = \frac{\overline{AO} \times \overline{AP}}{2} \qquad \tan x = \frac{\overline{AP}}{\overline{AO}} \Leftrightarrow \overline{AP} = \tan x$$

$$= \frac{1 \times \tan x}{2} = \frac{\tan x}{2} \qquad A_{[ABCD]} = 2 \times 1 = 2$$

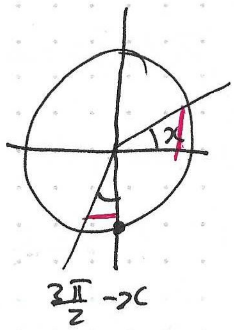
$$A_{[ODCBP]} = 2 - \frac{\tan x}{2}$$

logo $A_{[BCDQP]} = 2 - \dfrac{\tan x}{2} + \dfrac{\operatorname{sen} x}{2}$

c.q.m.

1.2

$\cos\left(\dfrac{3\pi}{2} - x\right) = -\dfrac{3}{5}$

$\cos\left(\dfrac{3\pi}{2} - x\right) =$  $= -\operatorname{sen} x$

Então $\cos\left(\dfrac{3\pi}{2} - x\right) = -\dfrac{3}{5} \Leftrightarrow$

$\Leftrightarrow -\operatorname{sen} x = -\dfrac{3}{5}$

$\Leftrightarrow \operatorname{sen} x = \dfrac{3}{5}$

$\operatorname{sen}^2 x + \cos^2 x = 1 \Leftrightarrow \cos^2 x = 1 - \operatorname{sen}^2 x$

$\Leftrightarrow \cos^2 x = 1 - \left(\dfrac{3}{5}\right)^2$

$\Leftrightarrow \cos^2 x = 1 - \dfrac{9}{25}$

$\Leftrightarrow \cos^2 x = \dfrac{16}{25}$, $x \in \left]0, \dfrac{\pi}{4}\right[$

$\cos x > 0$

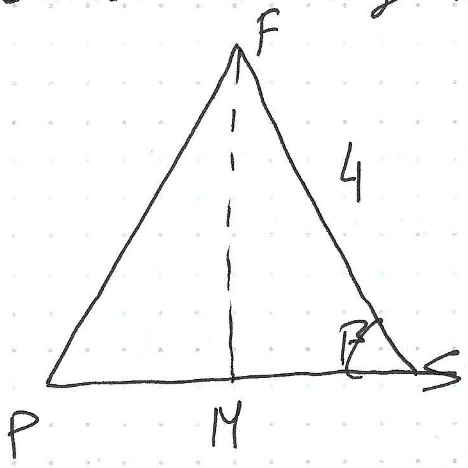
$\Leftrightarrow \cos x = \sqrt{\dfrac{16}{25}}$

$\Leftrightarrow \cos x = \dfrac{4}{5}$

Como $\tan x = \dfrac{\operatorname{sen} x}{\cos x} \Leftrightarrow \tan x = \dfrac{\frac{3}{5}}{\frac{4}{5}} \Leftrightarrow \tan x = \dfrac{3}{4}$

Assim $A_{[BCDQP]} = 2 - \dfrac{3/4}{2} + \dfrac{3/5}{2} = \dfrac{77}{40}$

2. Usando a sugestão



β é a amplitude de $F\hat{S}P$

M é ponto médio de $[PS]$

$$A_{[FSP]} = \frac{\overline{PS} \times \overline{FM}}{2}$$

$$\operatorname{sen}\beta = \frac{\overline{FM}}{4} \Leftrightarrow \overline{FM} = 4\operatorname{sen}\beta$$

$$\cos\beta = \frac{\overline{MS}}{4} \Leftrightarrow \overline{MS} = 4\cos\beta$$

como $\overline{PS} = 2\overline{MS}$ então

$\overline{PS} = 8\cos\beta$

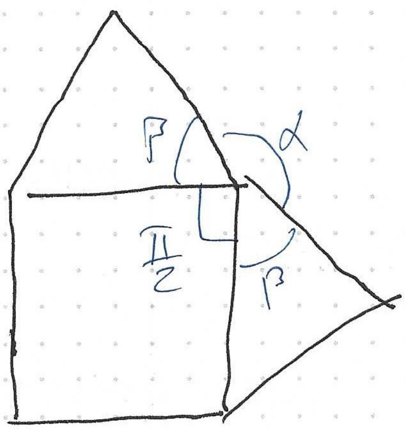
Assim

$$A_{[FSP]} = \frac{8\cos\beta \cdot 4\operatorname{sen}\beta}{2} = 16\cos\beta\operatorname{sen}\beta$$

como no enunciado $\operatorname{sen}(2\beta) = 2\operatorname{sen}\beta\cos\beta$

$$16\cos\beta\operatorname{sen}\beta = 8 \times 2\cos\beta\operatorname{sen}\beta =$$

$$= 8\operatorname{sen}(2\beta)$$



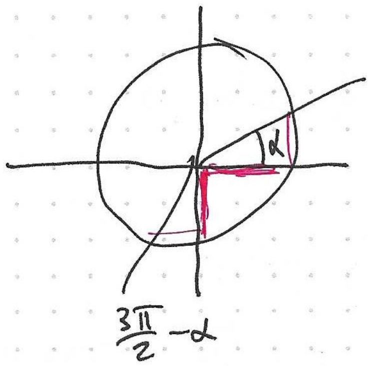
Pela observação da figura

$$\beta + \alpha + \beta + \frac{\pi}{2} = 2\pi$$

logo $\quad 2\beta = 2\pi - \frac{\pi}{2} - \alpha$

$$2\beta = \frac{3\pi}{2} - \alpha$$

logo $\quad 8 \operatorname{sen}(2\beta) = 8 \operatorname{sen}\left(\frac{3\pi}{2} - \alpha\right)$



$$\operatorname{sen}\left(\frac{3\pi}{2} - \alpha\right) = -\cos\alpha$$

$$8 \operatorname{sen}\left(\frac{3\pi}{2} - \alpha\right) = -8 \cos\alpha$$

Portanto a área lateral da pirâmide é $4 \times (-8\cos\alpha) = -32\cos\alpha$

c.q.m.