1 $\cos\alpha < 0$ nos 2º e 3º Q

$\cos\alpha$ é crescente nos 3º e 4º quadrantes

Opção C

2.1 $2P + 3\,\text{sen}\,\alpha = 1 + P \Leftrightarrow 3\,\text{sen}\,\alpha = 1 - P$

$\Leftrightarrow \text{sen}\,\alpha = \dfrac{1-P}{3}$

$\alpha \in \left]\dfrac{\pi}{2}, \pi\right[\quad \text{sen}\,\dfrac{\pi}{2} = 1 \quad \text{sen}\,\pi = 0$

$0 < \text{sen}\,\alpha < 1 \Leftrightarrow 0 < \dfrac{1-P}{3} < 1$

$\Leftrightarrow 0 < 1 - P < 3$

$\Leftrightarrow -1 < -P < 2$

$\Leftrightarrow -2 < P < 1$

$P \in \left]-2, 1\right[$

2.2 $\tan\alpha = -9p^2+6 \wedge \alpha \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right[$

Como $\alpha \in \left[\pi/4, \pi/2\right[$ então $1 \leq \tan\alpha < +\infty$

isto é, $\tan\alpha \geq 1$

logo $-9p^2+6 \geq 1 \Leftrightarrow -9p^2 \geq -5$

$$\Leftrightarrow 9p^2 \leq 5$$

$$\Leftrightarrow p^2 \leq \frac{5}{9}$$

$$\Leftrightarrow -\frac{\sqrt{5}}{3} \leq p \leq \frac{\sqrt{5}}{3}$$

C.A.

$p^2 = \frac{5}{9} \Leftrightarrow$

$p = \pm\sqrt{\frac{5}{9}}$

$p = \pm\frac{\sqrt{5}}{3}$

$$p \in \left[-\frac{\sqrt{5}}{3}, \frac{\sqrt{5}}{3}\right]$$

2.3 $\cos\alpha = 3-p \wedge \alpha \in [0, \pi]$

Como $\alpha \in [0, \pi]$ então $-1 \leq \cos\alpha \leq 1$

$-1 \leq \cos\alpha \leq 1 \Leftrightarrow -1 \leq 3-p \leq 1$

$$\Leftrightarrow -4 \leq -p \leq -2$$

$$\Leftrightarrow 2 \leq p \leq 4$$

$$p \in [2, 4]$$

2.4 $\operatorname{sen} x = -16p^2 + 8 \wedge \alpha \in \left[\frac{\pi}{6}, \frac{\pi}{2}\right[$

Como $\alpha \in \left[\frac{\pi}{6}, \frac{\pi}{2}\right[$ então $\frac{1}{2} \leq \operatorname{sen}\alpha < 1$

$$\frac{1}{2} \leq \operatorname{sen}\alpha < 1 \Leftrightarrow \frac{1}{2} \leq -16p^2 + 8 < 1$$

$$\Leftrightarrow 1 \leq -32p^2 + 16 < 2$$

$$\Leftrightarrow -15 \leq -32p^2 < -14$$

$$\Leftrightarrow 14 \leq 32p^2 < 15$$

$$\Leftrightarrow \frac{14}{32} \leq p^2 < \frac{15}{32}$$

$$\Leftrightarrow \frac{7}{16} \leq p^2 < \frac{15}{32}$$

$$\Leftrightarrow p^2 < \frac{15}{32} \wedge p^2 \geq \frac{7}{16}$$

C.A.

$p^2 = \frac{15}{32}$

$p = \pm\sqrt{\frac{15}{32}}$

$p = \pm\frac{1}{4}\sqrt{\frac{15}{2}}$

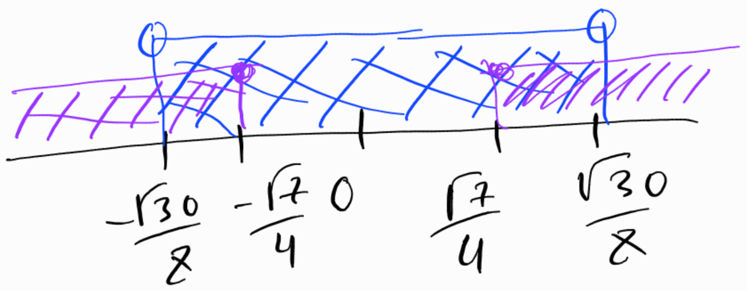
$p = \pm\frac{1}{8}\sqrt{30}$

$p^2 = \frac{7}{16}$

$p = \pm\frac{\sqrt{7}}{4}$

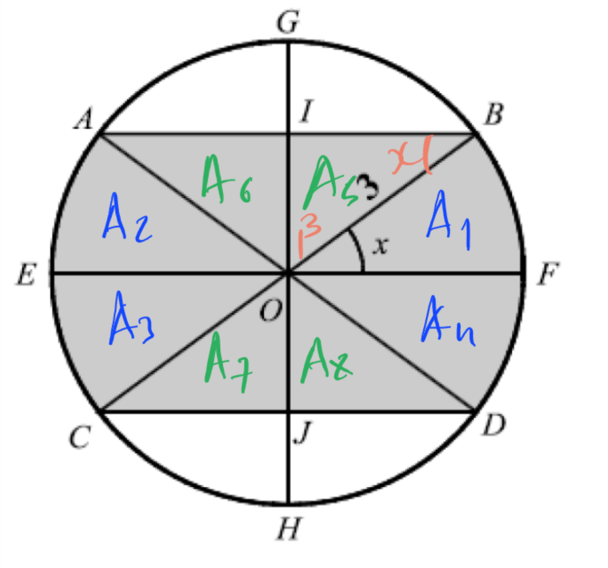
$$\Leftrightarrow p \in \left]-\frac{\sqrt{30}}{8}, \frac{\sqrt{30}}{8}\right[\wedge$$

$$\wedge\ p \in \left]-\infty, -\frac{\sqrt{7}}{4}\right] \cup \left[\frac{\sqrt{7}}{4}, +\infty\right[$$



$$x \in \left]-\frac{\sqrt{30}}{8}, -\frac{\sqrt{7}}{4}\right] \cup \left[\frac{\sqrt{7}}{4}, \frac{\sqrt{30}}{8}\right[$$

3.



A_1, A_2, A_3 e A_4 são setores circulares

Área de um setor circular é $\frac{\alpha r^2}{2}$

(Formulário de exame)

A_5, A_6, A_7 e A_8 são triângulos retângulos

$$A_{sombreada} = 4A_1 + 4A_5$$

$$A_1 = \frac{\alpha r^2}{2} = \frac{x \cdot 3^2}{2} = \frac{x9}{2}$$

$$4A_1 = 4 \cdot \frac{x9}{2} = 18x$$

$$A_5 = \frac{\overline{IB} \times \overline{OI}}{2}$$

$$A_5 = \frac{3\cos x \cdot 3\operatorname{sen} x}{2}$$

$$A_5 = \frac{9\cos x \operatorname{sen} x}{2}$$

$$4A_5 = 18\cos x \operatorname{sen} x$$

$$A_{som} = 18x + 18\cos x \operatorname{sen} x$$

$$= 18(x + \cos x \operatorname{sen} x) \quad \text{c.q.m.}$$

C. A.

$$\beta = \frac{\pi}{2} - x$$

$$\operatorname{sen}\left(\frac{\pi}{2} - x\right) = \frac{\overline{IB}}{3}$$

$$3\cos x = \overline{IB}$$

$$\operatorname{sen} x = \frac{\overline{OI}}{3}$$

$$\overline{OI} = 3\operatorname{sen} x$$

4.1 $D_f = \mathbb{R}$

D'_f: $-1 \le \cos\left(3x+\frac{\pi}{6}\right) \le 1$

$-1 \le -\cos\left(3x+\frac{\pi}{6}\right) \le 1$

$-2 \le -2\cos\left(3x+\frac{\pi}{6}\right) \le 2$

$-1 \le 1-2\cos\left(3x+\frac{\pi}{6}\right) \le 3$

$D'_f = [-1, 3]$

4.2 Seja P o período fundamental

$x \in D_f$, então $x+P \in D_f$, porque $D_f = \mathbb{R}$

$\forall x \in \mathbb{R},\ f(x+P) = f(x)$

$\cancel{1}\cancel{-2}\cos\left(3(x+P)+\frac{\pi}{6}\right) = \cancel{1}\cancel{-2}\cos\left(3x+\frac{\pi}{6}\right)$

$\cos x\left(3x+3P+\frac{\pi}{6}\right) = \cos\left(3x+\frac{\pi}{6}\right)$

$\cos x\left(3x+\frac{\pi}{6}+3P\right) = \cos\left(3x+\frac{\pi}{6}\right)$

Assim, $3P = 2\pi \Leftrightarrow P = \frac{2\pi}{3}$

Logo f é periódica de período fundamental $P = \frac{2\pi}{3}$

5. $f(x) = -\sqrt{3} + 2\,\text{sen}\,x$

5.1 $f\left(\frac{4\pi}{3}\right) - f(\pi) + \sqrt{3}\, f\left(\frac{\pi}{6}\right) =$

$= -\sqrt{3} + 2\,\text{sen}\left(\frac{4\pi}{3}\right) + \sqrt{3} - 2\,\text{sen}(\pi) + \sqrt{3}\left(-\sqrt{3} + 2\,\text{sen}\left(\frac{\pi}{6}\right)\right)$

C.A. $\text{sen}\left(\frac{4\pi}{3}\right) = \text{sen}\left(\pi + \frac{\pi}{3}\right) = -\text{sen}\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

$\text{sen}(\pi) = 0$

$\text{sen}\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$= -\sqrt{3} - \cancel{2}\frac{\sqrt{3}}{\cancel{2}} + \sqrt{3} - 2 \times 0 + \sqrt{3}\left(-\sqrt{3} + \cancel{2} \times \frac{1}{\cancel{2}}\right) =$

$= \cancel{-\sqrt{3}} \cancel{-\sqrt{3}} \cancel{+\sqrt{3}} - 3 \cancel{+\sqrt{3}} = -3$

5.2 $D_f = \mathbb{R}$

D'_f: $-1 \leq \text{sen}\,x \leq 1$

$-2 \leq 2\,\text{sen}\,x \leq 2$

$-\sqrt{3} - 2 \leq -\sqrt{3} + 2\,\text{sen}\,x \leq 2 - \sqrt{3}$

$D'_f = [-2-\sqrt{3}, 2-\sqrt{3}]$

5.3 $f(-x) = -\sqrt{3} + 2\,\text{sen}(-x) =$

$= -\sqrt{3} - 2\,\text{sen}(x) \neq -f(x)$

logo f não é par nem ímpar

5.4

$\cancel{-\sqrt{3}} + 2\,\text{sen}\,\alpha = 1 \cancel{-\sqrt{3}} \Leftrightarrow$

$\Leftrightarrow \text{sen}\,\alpha = \frac{1}{2} \Leftrightarrow \text{sen}\,\alpha = \text{sen}\left(\frac{\pi}{6}\right)$

$\Leftrightarrow \alpha = \frac{\pi}{6} + 2k\pi \vee \alpha = \pi - \frac{\pi}{6} + 2k\pi,\ k \in \mathbb{Z}$

$\Leftrightarrow \alpha = \frac{\pi}{6} + 2k\pi \vee \alpha = \frac{5\pi}{6} + 2k\pi,\ k \in \mathbb{Z}$

$k = 0: \quad \alpha = \frac{\pi}{6} \vee \alpha = \frac{5\pi}{6},\ \alpha \in \left]\frac{\pi}{2}, \pi\right[$

$\alpha = \frac{5\pi}{6} = \pi - \frac{\pi}{6}$

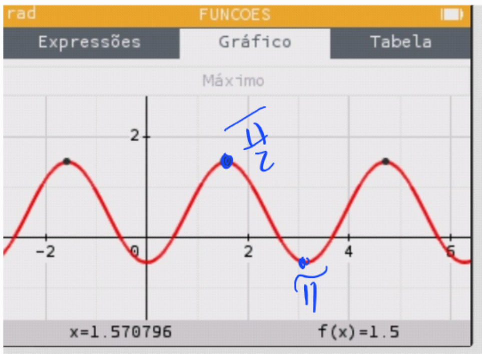
$\tan(3\pi - \alpha) - \text{sen}(\pi - \alpha)$

$\tan\left(3\pi - \left(\pi - \frac{\pi}{6}\right)\right) - \text{sen}\left(\pi - \left(\pi - \frac{\pi}{6}\right)\right)$

$\tan\left(2\pi + \frac{\pi}{6}\right) - \text{sen}\left(-\frac{\pi}{6}\right) =$

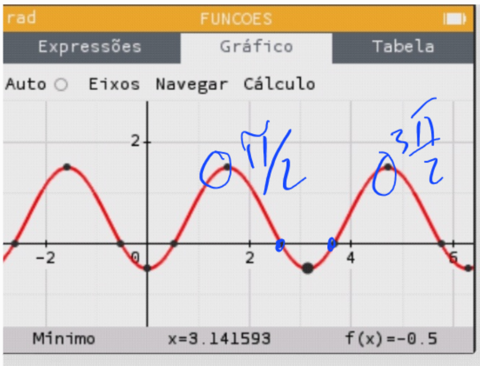
$\tan\left(\frac{\pi}{6}\right) + \text{sen}\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{3} + \frac{1}{2} = \frac{2\sqrt{3} + 3}{6}$

6.1.1



$$x \in \left[\frac{\pi}{2}, \pi\right]$$

6.1.2



$$x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$$

6.2

D'_g

$$-1 \leq \cos 2x \leq 1$$

$$-1 \leq -\cos 2x \leq 1$$

$$\frac{1}{2} - 1 \leq \frac{1}{2} - \cos 2x \leq 1 + \frac{1}{2}$$

$$-\frac{1}{2} \leq \frac{1}{2} - \cos 2x \leq \frac{3}{2}$$

$$D'_g = \left[-\frac{1}{2}, \frac{3}{2}\right]$$

6.3 $g(-x)=\frac{1}{2}-\cos(2(-x))=\frac{1}{2}-\cos(-2x)=$

$\cos(-\alpha)=\cos\alpha$

$=\frac{1}{2}-\cos 2x=g(x)$

logo g é par

6.4 $g(x+P)=g(x) \Leftrightarrow \cancel{\frac{1}{2}-}\cos(2(x+P))=\cancel{\frac{1}{2}-}\cos(2x)$

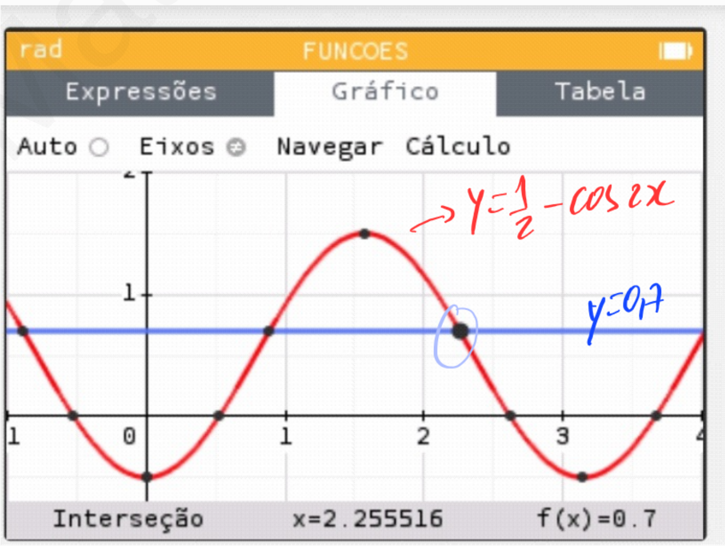
$\Leftrightarrow \cos(2x+2P)=\cos(2x)$

período fundamental da função cosseno é 2π.

Assim, $2P=2\pi \Leftrightarrow P=\pi$

O período da função g é π

6.5



$x\approx 2{,}26$

7.

π radianos ———— 180^{0}

3 radianos ———— x

$$x = \frac{3 \times 180}{\pi} \approx 171,9$$

OPÇÃO : B

8.1

$$\frac{\tan^2\alpha - 1}{1+\tan^2\alpha} = \frac{\frac{\text{sen}^2\alpha}{\cos^2\alpha} - 1}{\frac{1}{\cos^2\alpha}} =$$

$$\frac{\frac{\text{sen}^2\alpha - \cos^2\alpha}{\cancel{\cos^2\alpha}}}{\frac{1}{\cancel{\cos^2\alpha}}} = \text{sen}^2\alpha - \cos^2\alpha$$

$\cos^2\alpha = 1 - \text{sen}^2\alpha$

$$= \text{sen}^2\alpha - (1 - \text{sen}^2\alpha) = 2\text{sen}^2\alpha - 1$$

c. q. m.

8.2

$$A(-x) = 2\text{sen}^2(-\alpha) - 1$$

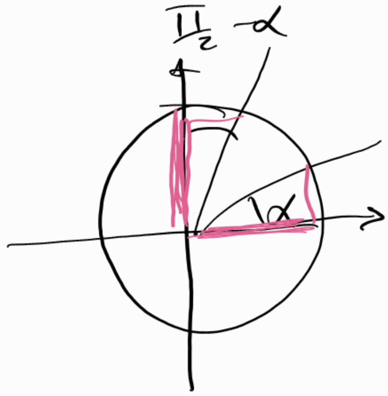
$$= 2(\text{sen}(-\alpha))^2 - 1 =$$

$$= 2(-\text{sen}\,\alpha)^2 - 1 =$$

$$= 2(\text{sen}\alpha)^2 - 1 = 2\,\text{sen}^2\alpha - 1$$

$= A(x)$. logo A é par

8.3 $\operatorname{sen}\left(\frac{\pi}{2}-\alpha\right)=\cos\alpha$



$\cos\alpha=\frac{2\sqrt{5}}{5}$

$\operatorname{sen}^2\alpha+\cos^2\alpha=1$

$\Leftrightarrow \operatorname{sen}^2\alpha+\left(\frac{2\sqrt{5}}{5}\right)^2=1 \Leftrightarrow$

$\Leftrightarrow \operatorname{sen}^2\alpha+\frac{4\times5}{25}=1 \Leftrightarrow$

$\Leftrightarrow \operatorname{sen}^2\alpha+\frac{4}{5}=1 \Leftrightarrow \operatorname{sen}^2\alpha=1-\frac{4}{5}$

$\Leftrightarrow \operatorname{sen}^2\alpha=\frac{1}{5} \Leftrightarrow \operatorname{sen}\alpha=\pm\sqrt{\frac{1}{5}}$

$\Leftrightarrow \operatorname{sen}\alpha=\pm\frac{\sqrt{5}}{5}$

Como $\alpha\in\left]\pi,2\pi\right[$ então $\operatorname{sen}\alpha<0$

logo $\operatorname{sen}\alpha=-\frac{\sqrt{5}}{5}$

Assim $2\operatorname{sen}^2\alpha-1=2\times\left(-\frac{\sqrt{5}}{5}\right)^2-1$

$=2\times\frac{5}{25}-1=\frac{2}{5}-1=-\frac{3}{5}$

9.1 $f(22) = 14 + 6 \operatorname{sen}\left[\frac{\pi(22+z)}{6}\right]$

$= 14 + 6 \operatorname{sen}(5\pi)$

$= 14 + 6 \times 0 = 14$

Às 22h, a distância é de 14m

9.2

$-1 \leq \operatorname{sen}\left[\frac{\pi(x+z)}{6}\right] \leq 1$

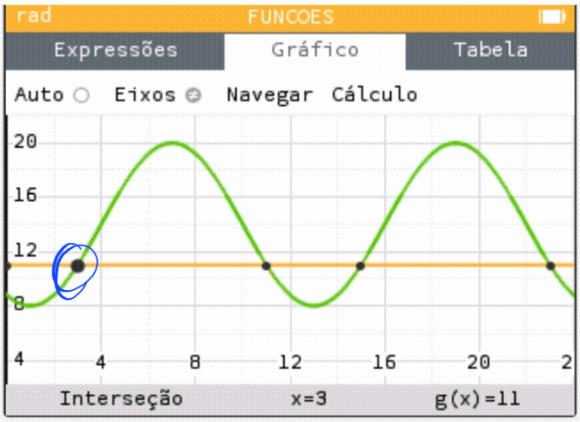
$-6 \leq 6 \operatorname{sen}\left[\frac{\pi(x+z)}{6}\right] \leq 6$

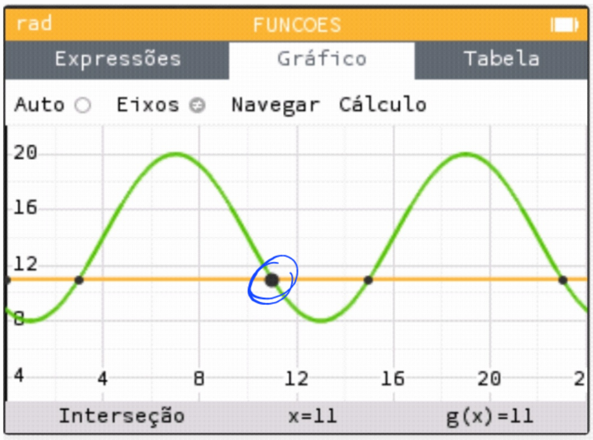
$14-6 \leq 14 + 6 \operatorname{sen}\left[\frac{\pi(x+z)}{6}\right] \leq 6+14$

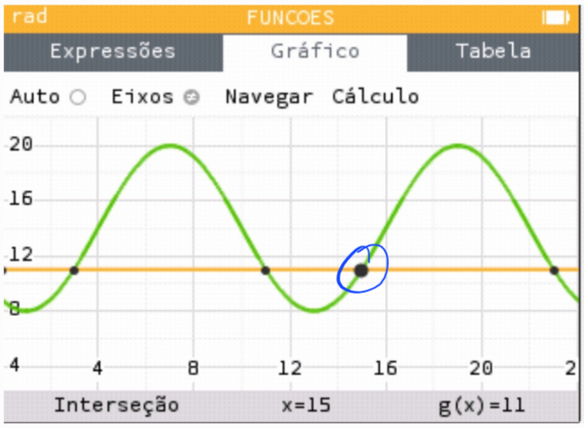
$8 \leq 14 + 6 \operatorname{sen}\left[\frac{\pi(x+z)}{6}\right] \leq 20$

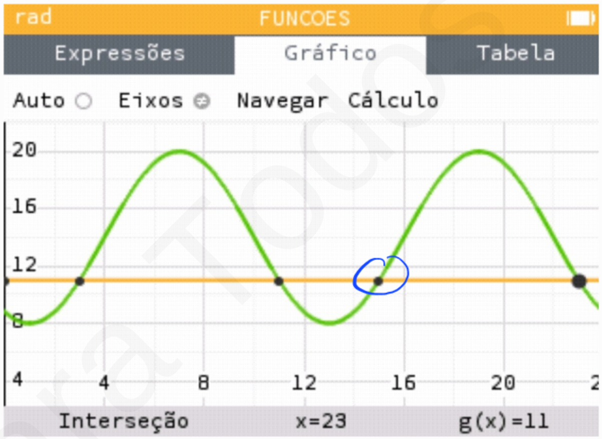
A distância varia entre os 8 m e os 20 m.

9.3









Por observação dos gráficos a distância igual a 11 metros aconteceu às 3 horas, 11 horas, 15 horas e 23 horas.