1. Determina o domínio, os zeros e estuda o sinal de cada uma das funções analíticas:

a) $f(x) = \frac{x^2+2x+1}{x+1}$

$f(x) = \frac{x^2+2x+1}{x+1}$

- $D_f = \{x \in \mathbb{R}: x+1 \neq 0\} = \mathbb{R} \setminus \{-1\}$
- $f(x) = 0 \Leftrightarrow \frac{x^2+2x+1}{x+1} = 0$

$\Leftrightarrow x^2+2x+1=0 \wedge x+1 \neq 0$

$\Leftrightarrow (x+1)^2 = 0 \wedge x \neq -1$

$\Leftrightarrow x=-1 \wedge x \neq -1$, que é uma condição impossível.

Logo, f não tem zeros.

$f(x) = \frac{x^2+2x+1}{x+1} = \frac{(x+1)^2}{x+1} = x+1$

Assim, $f(x) > 0 \Leftrightarrow x \in \left]-1, +\infty\right[$ e
$f(x) < 0 \Leftrightarrow x \in \left]-\infty, -1\right[$.

b) $h(x) = \frac{x^2-2x+1}{x^2+5x+6}$

$h(x) = \frac{x^2-2x+1}{x^2+5x+6}$

- $D_h = \{x \in \mathbb{R}: x^2+5x+6 \neq 0\} = \mathbb{R} \setminus \{-3, -2\}$

Cálculo auxiliar

$x^2+5x+6=0 \Leftrightarrow x = \frac{-5 \pm \sqrt{25-24}}{2}$

$\Leftrightarrow x = \frac{-5 \pm 1}{2}$

$\Leftrightarrow x=-3 \vee x=-2$

- $h(x) = 0 \Leftrightarrow \frac{x^2-2x+1}{x^2+5x+6} = 0$

$\Leftrightarrow x^2-2x+1=0 \wedge x^2+5x+6 \neq 0$

$\Leftrightarrow (x-1)^2=0 \wedge x \neq -3 \wedge x \neq -2$

$\Leftrightarrow x-1=0 \wedge x \neq -3 \wedge x \neq -2$

$\Leftrightarrow x=1$

x	$-\infty$	-3		-2		1	$+\infty$
x^2-2x+1	+	+	+	+	+	0	+
x^2+5x+6	+	0	−	0	+	+	+
$h(x)$	+	n.d.	−	n.d.	+	0	+

Assim:
$h(x) > 0 \Leftrightarrow x \in \left]-\infty, -3\right[\cup \left]-2, 1\right[\cup \left]1, +\infty\right[$ e
$h(x) < 0 \Leftrightarrow x \in \left]-3, -2\right[$

c) $i(x) = \frac{1}{(x-2)^2} - \frac{5}{x^2-4}$

$i(x) = \frac{1}{(x-2)^2} - \frac{5}{x^2-4}$

- $D_i = \{x \in \mathbb{R}: (x-2)^2 \neq 0 \ \wedge \ x^2 - 4 \neq 0\} = \mathbb{R} \setminus \{-2, 2\}$
- $i(x) = 0 \Leftrightarrow \frac{1}{(x-2)^2} - \frac{5}{x^2-4} = 0$

$\Leftrightarrow \frac{x+2-5(x-2)}{(x-2)^2(x+2)} = 0$

$\Leftrightarrow \frac{-4x+12}{(x-2)^2(x+2)} = 0$

$\Leftrightarrow -4x + 12 = 0 \ \wedge \ (x-2)^2(x+2) \neq 0$

$\Leftrightarrow x = 3$

Logo, 3 é um zero de *i*.

x	$-\infty$	-2		2		3	$+\infty$
$-4x+12$	+	+	+	+	+	0	−
$(x-2)^2$	+	+	+	0	+	+	+
$x+2$	−	0	+	+	+	+	+
$i(x)$	−	n.d.	+	0	+	0	−

Assim:

$i(x) < 0 \Leftrightarrow x \in]-\infty, -2[\cup]3, +\infty[$

e $i(x) > 0 \Leftrightarrow x \in]-2, 2[\cup]2, 3[$

d) $k(x) = \frac{x^3-1}{x+1} + \frac{1}{x} - \frac{x^3+1}{x-1}$

$k(x) = \frac{x^3-1}{x+1} + \frac{1}{x} - \frac{x^3+1}{x-1}$

- $D_k = \{x \in \mathbb{R}: x+1 \neq 0 \ \wedge \ x \neq 0 \ \wedge \ x-1 \neq 0\} =$
$= \mathbb{R} \setminus \{-1, 0, 1\}$
- $k(x) = 0 \Leftrightarrow \frac{x^3-1}{x+1} + \frac{1}{x} - \frac{x^3+1}{x-1} = 0$

$\Leftrightarrow \frac{(x^3-1)x(x-1) + (x-1)(x+1) - (x^3+1)x(x+1)}{(x+1)x(x-1)} = 0$

$\Leftrightarrow \frac{-2x^4 - x^2 - 1}{(x+1)x(x-1)} = 0$

$\Leftrightarrow -2x^4 - x^2 - 1 = 0 \ \wedge \ (x+1)x(x-1) \neq 0$

$\Leftrightarrow x^2 = \frac{1 \pm \sqrt{1-8}}{-4}$, que é uma equação impossível em $\mathbb{R}$.

Logo, *k* não tem zeros.

x	$-\infty$	-1		0		1	$+\infty$
$-2x^4-x^2-1$	−	−	−	−	−	−	−
$x+1$	−	0	+	+	+	+	+
x	−	−	−	0	+	+	+
$x-1$	−	−	−	−	−	0	+
$k(x)$	+	n.d.	−	n.d.	+	n.d	−

Assim:

$k(x) > 0 \Leftrightarrow x \in]-\infty, -1[\cup]0, 1[$

e $k(x) < 0 \Leftrightarrow x \in]-1, 0[\cup]1, +\infty[$

2. Resolve as seguintes equações:

a) $x+3=\frac{3}{x-1}$

$$x+3=\frac{3}{x-1} \Leftrightarrow x+3-\frac{3}{x-1}=0$$
$$\Leftrightarrow \frac{x^2+3x-x-3-3}{x-1}=0$$
$$\Leftrightarrow \frac{x^2+2x-6}{x-1}=0$$
$$\Leftrightarrow x^2+2x-6=0 \wedge x-1\neq 0$$
$$\Leftrightarrow x=\frac{-2\pm\sqrt{4+24}}{2} \wedge x\neq 1$$
$$\Leftrightarrow x=\frac{-2\pm 2\sqrt{7}}{2} \wedge x\neq 1$$
$$\Leftrightarrow x=-1-\sqrt{7} \vee x=-1+\sqrt{7}$$

$\text{C.S.}=\{-1-\sqrt{7}, -1+\sqrt{7}\}$

b) $\frac{x-2}{2}+\frac{x+1}{2-x}=\frac{1}{4}$

$$\frac{x-2}{2}+\frac{x+1}{2-x}=\frac{1}{4} \Leftrightarrow \frac{x-2}{2}+\frac{x+1}{2-x}-\frac{1}{4}=0$$
$$\Leftrightarrow \frac{2(x-2)(2-x)+4(x+1)-(2-x)}{4(2-x)}=0$$
$$\Leftrightarrow \frac{-2x^2+13x-6}{4(2-x)}=0$$
$$\Leftrightarrow -2x^2+13x-6=0 \wedge 4(2-x)\neq 0$$
$$\Leftrightarrow x=\frac{-13\pm\sqrt{169-48}}{-4} \wedge x\neq 2$$
$$\Leftrightarrow \left(x=\frac{1}{2} \vee x=6\right) \wedge x\neq 2$$
$$\Leftrightarrow x=\frac{1}{2} \vee x=6$$

$\text{C.S.}=\left\{\frac{1}{2}, 6\right\}$

c) $\frac{3}{x}-\frac{2}{x-2}=-1$

$$\frac{3}{x}-\frac{2}{x-2}=-1 \Leftrightarrow \frac{3}{x}-\frac{2}{x-2}+1=0$$
$$\Leftrightarrow \frac{3x-6-2x+x^2-2x}{x(x-2)}=0$$
$$\Leftrightarrow \frac{x^2-x-6}{x(x-2)}=0$$
$$\Leftrightarrow x^2-x-6=0 \wedge x(x-2)\neq 0$$
$$\Leftrightarrow x=\frac{1\pm\sqrt{1+24}}{2} \wedge x\neq 0 \wedge x\neq 2$$
$$\Leftrightarrow x=\frac{1\pm 5}{2} \wedge x\neq 0 \wedge x\neq 2$$
$$\Leftrightarrow x=3 \vee x=-2$$

$\text{C.S.}=\{-2, 3\}$

d) $\frac{2}{x^2+2x}-\frac{1}{x}=\frac{x}{x+2}$

$$\frac{2}{x^2+2x}-\frac{1}{x}=\frac{x}{x+2} \Leftrightarrow \frac{2}{x^2+2x}-\frac{1}{x}-\frac{x}{x+2}=0$$
$$\Leftrightarrow \frac{2-x-2-x^2}{x^2+2x}=0$$
$$\Leftrightarrow \frac{-x^2-x}{x^2+2x}=0$$
$$\Leftrightarrow \frac{x(-x-1)}{x(x+2)}=0$$
$$\Leftrightarrow \frac{-x-1}{x+2}=0$$
$$\Leftrightarrow -x-1=0 \wedge x+2\neq 0$$
$$\Leftrightarrow x=-1$$

$\text{C.S.}=\{-1\}$

e) $\frac{x}{x+5}-\frac{x}{x-2}=\frac{3}{x^2+3x-10}$

$$\frac{x}{x+5}-\frac{x}{x-2}=\frac{3}{x^2+3x-10}$$

$$\Leftrightarrow \frac{x}{x+5}-\frac{x}{x-2}-\frac{3}{x^2+3x-10}=0$$

$$\Leftrightarrow \frac{x^2-2x-x^2-5x-3}{(x+5)(x-2)}=0$$

$$\Leftrightarrow \frac{-7x-3}{(x+5)(x-2)}=0$$

$$\Leftrightarrow -7x-3=0 \wedge (x+5)(x-2)\neq 0$$

$$\Leftrightarrow x=-\frac{3}{7}$$

Cálculo auxiliar

$$x^2+3x-10=0 \Leftrightarrow x=\frac{-3\pm\sqrt{9+40}}{2}$$

$$\Leftrightarrow x=\frac{-3\pm 7}{2}$$

$$\Leftrightarrow x=-5 \vee x=2$$

$$\text{C.S.}=\left\{-\frac{3}{7}\right\}$$

f) $\frac{3}{x-1}-\frac{10}{x^2+3x-4}=\frac{x}{x+4}$

$$\frac{3}{x-1}-\frac{10}{x^2+3x-4}=\frac{x}{x+4}$$

$$\Leftrightarrow \frac{3}{x-1}-\frac{10}{x^2+3x-4}-\frac{x}{x+4}=0$$

$$\Leftrightarrow \frac{3x+12-10-x^2+x}{(x-1)(x+4)}=0$$

$$\Leftrightarrow \frac{-x^2+4x+2}{(x-1)(x+4)}=0$$

$$\Leftrightarrow -x^2+4x+2=0 \wedge (x-1)(x+4)\neq 0$$

$$\Leftrightarrow x=\frac{-4\pm\sqrt{16+8}}{-2} \wedge x\neq 1 \wedge x\neq -4$$

$$\Leftrightarrow x=\frac{4\pm 2\sqrt{6}}{2} \wedge x\neq 1 \wedge x\neq -4$$

$$\Leftrightarrow x=2-\sqrt{6} \vee x=2+\sqrt{6}$$

Cálculo auxiliar

$$x^2+3x-4=0 \Leftrightarrow x=\frac{-3\pm\sqrt{9+16}}{2}$$

$$\Leftrightarrow x=\frac{-3\pm 5}{2}$$

$$\Leftrightarrow x=1 \vee x=-4$$

$$\text{C.S.}=\left\{2-\sqrt{6}, 2+\sqrt{6}\right\}$$

g) $\frac{x^2+3x}{x+1}=3$

$$\frac{x^2+3x}{x+1}=3 \Leftrightarrow \frac{x^2+3x}{x+1}-3=0 \Leftrightarrow \frac{x^2+3x-3x-3}{x+1}=0 \Leftrightarrow \frac{x^2-3}{x+1}=0 \Leftrightarrow$$

$$\Leftrightarrow x^2-3=0 \wedge x+1\neq 0 \Leftrightarrow x^2=3 \wedge x\neq -1 \Leftrightarrow x=\pm\sqrt{3}$$

Conjunto-solução: $\{-\sqrt{3}, \sqrt{3}\}$

h) $\frac{3-5x}{x^2-9}=\frac{x-1}{3-x}$

$$\frac{3-5x}{x^2-9}=\frac{x-1}{3-x} \Leftrightarrow \frac{3-5x}{(x-3)(x+3)}=\frac{-x+1}{x-3} \Leftrightarrow \frac{3-5x}{(x-3)(x+3)}-\underset{(x+3)}{\frac{-x+1}{x-3}}=0 \Leftrightarrow$$

$$\Leftrightarrow \frac{3-5x+x^2-x+3x-3}{(x-3)(x+3)}=0 \Leftrightarrow \frac{x^2-3x}{(x-3)(x+3)}=0 \Leftrightarrow \frac{x(x-3)}{(x-3)(x+3)}=0 \Leftrightarrow \frac{x}{x+3}=0 \Leftrightarrow$$

$$\Leftrightarrow x=0 \wedge x\neq -3 \wedge x\neq 3 \Leftrightarrow x=0 \wedge x\neq -3 \wedge x\neq 3 \Leftrightarrow x=0$$

Conjunto-solução: $\{0\}$

i) $\frac{x}{x-2} = \frac{4}{6x-x^2-8}$

$\frac{x}{x-2} = \frac{4}{6x-x^2-8} \underset{(*)}{\Leftrightarrow} \frac{x}{x-2} = \frac{4}{-(x-4)(x-2)} \Leftrightarrow \frac{x}{\underset{(x-4)}{x-2}} + \frac{4}{(x-4)(x-2)} = 0 \Leftrightarrow$

$\Leftrightarrow \frac{x^2-4x+4}{(x-4)(x-2)} = 0 \Leftrightarrow \frac{(x-2)^2}{(x-4)(x-2)} = 0 \Leftrightarrow \frac{x-2}{x-4} = 0 \Leftrightarrow$

$\Leftrightarrow x-2=0 \wedge x \neq 2 \wedge x \neq 4 \Leftrightarrow x=2 \wedge x \neq 2 \wedge x \neq 4 \rightarrow$ equação impossível

Conjunto-solução: $\emptyset$

$(*) -x^2+6x-8=0 \Leftrightarrow x = \frac{-6\pm\sqrt{36-4\times(-1)\times(-8)}}{2\times(-1)} \Leftrightarrow x = \frac{-6\pm2}{-2} \Leftrightarrow$

$\Leftrightarrow x=2 \vee x=4$; $-x^2+6x-8 = -(x-4)(x-2)$

j) $x = \frac{3x-2}{x^2}$

$x = \frac{3x-2}{x^2} \Leftrightarrow x - \frac{3x-2}{x^2} = 0 \Leftrightarrow \frac{x^3-3x+2}{x^2} = 0 \Leftrightarrow x^3-3x+2=0 \wedge x^2 \neq 0 \Leftrightarrow$

$\underset{(*)}{\Leftrightarrow} (x-1)^2(x+2)=0 \wedge x \neq 0 \Leftrightarrow (x-1=0 \vee x+2=0) \wedge x \neq 0 \Leftrightarrow$

$\Leftrightarrow (x=1 \vee x=-2) \wedge x \neq 0 \Leftrightarrow x=1 \vee x=-2$

Conjunto-solução: $\{-2, 1\}$

(*)

	1	0	−3	2
1		1	1	−2
	1	1	−2	0
1		1	2	
	1	2	0	

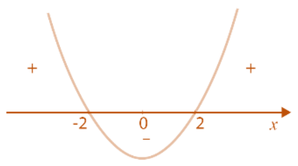
3. Resolve as seguintes inequações:

a) $\frac{2}{x^2-4} \leq 0$

$\frac{-2}{x^2-4} \leq 0 \Leftrightarrow x^2-4>0$

Cálculo auxiliar

$x^2-4=0 \Leftrightarrow x=2 \vee x=-2$



C.S. $=]-\infty, -2[\cup]2, +\infty[$

b) $\frac{x-1}{-2x+1} \geq 0$

$\frac{x-1}{-2x+1} \geq 0$

x	$-\infty$	$\frac{1}{2}$		1	$+\infty$
$x-1$	−	−	−	0	+
$-2x+1$	+	0	−	−	−
$\frac{x-1}{-2x+1}$	−	n.d.	+	0	−

C.S. $= \left]\frac{1}{2}, 1\right]$

c) $\frac{x^2+4x+3}{x^2-5x+6} \leq 0$

$\frac{x^2+4x+3}{x^2-5x+6} \leq 0$

Cálculos auxiliares

$x^2+4x+3=0 \Leftrightarrow x=\frac{-4\pm\sqrt{16-12}}{2}$

$\Leftrightarrow x=\frac{-4\pm 2}{2}$

$\Leftrightarrow x=-3 \vee x=-1$

$x^2-5x+6=0 \Leftrightarrow x=\frac{5\pm\sqrt{25-24}}{2}$

$\Leftrightarrow x=\frac{5\pm 1}{2}$

$\Leftrightarrow x=3 \vee x=2$

x	$-\infty$	-3		-1		2		3	$+\infty$
x^2+4x+3	+	0	−	0	+	+	+	+	+
x^2-5x+6	+	+	+	+	+	0	−	0	+
$\frac{x^2+4x+3}{x^2-5x+6}$	+	0	−	0	+	n.d.	−	n.d.	+

C.S. = $[-3, -1] \cup]2, 3[$

d) $\frac{x+1}{2x+1} < \frac{1}{x} + 2$

$\frac{x+1}{2x+1} < \frac{1}{x} + 2 \Leftrightarrow \frac{x+1}{2x+1} - \frac{1}{x} - 2 < 0$

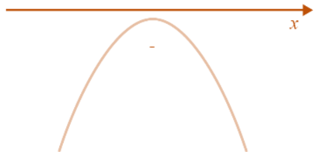
$\Leftrightarrow \frac{x^2+x-2x-1-4x^2-2x}{x(2x+1)} < 0$

$\Leftrightarrow \frac{-3x^2-3x-1}{x(2x+1)} < 0$

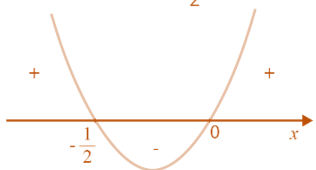
$\Leftrightarrow x(2x+1) > 0$

Cálculos auxiliares

$-3x^2-3x-1=0 \Leftrightarrow x=\frac{3\pm\sqrt{9-12}}{-6}$, que é uma equação impossível em $\mathbb{R}$.



$x(2x+1)=0 \Leftrightarrow x=0 \vee x=-\frac{1}{2}$



C.S. = $\left]-\infty, -\frac{1}{2}\right[\cup]0, +\infty[$

e) $\frac{3}{x} - \frac{2}{x-2} \leq -1$

$\frac{3}{x} - \frac{2}{x-2} \leq -1 \Leftrightarrow \frac{3}{x} - \frac{2}{x-2} + 1 \leq 0$

$\Leftrightarrow \frac{3x-6-2x+x^2-2x}{x(x-2)} \leq 0$

$\Leftrightarrow \frac{x^2-x-6}{x(x-2)} \leq 0$

Cálculo auxiliar

$x^2-x-6=0 \Leftrightarrow x=\frac{1\pm\sqrt{1+24}}{2}$

$\Leftrightarrow x=\frac{1\pm 5}{2}$

$\Leftrightarrow x=3 \vee x=-2$

x	$-\infty$	-2		0		2		3	$+\infty$
x^2-x-6	+	0	−	−	−	−	−	0	+
$x(x-2)$	+	+	+	0	−	0	+	+	+
$\frac{x^2-x-6}{x(x-2)}$	+	0	−	n.d.	+	n.d.	−	0	+

C.S. = $[-2, 0[\cup]2, 3]$

f) $1 - \frac{1}{x+1} \geq \frac{4}{x^2+x}$

$1 - \frac{1}{x+1} \geq \frac{4}{x^2+x} \Leftrightarrow 1 - \frac{1}{x+1} - \frac{4}{x^2+x} \geq 0$

$\Leftrightarrow \frac{x^2+x-x-4}{x^2+x} \geq 0$

$\Leftrightarrow \frac{x^2-4}{x^2+x} \geq 0$

Cálculos auxiliares

$x^2-4=0 \Leftrightarrow x=2 \vee x=-2$

$x^2+x=0 \Leftrightarrow x(x+1)=0 \Leftrightarrow x=0 \vee x=-1$

x	$-\infty$	-2		-1		0		2	$+\infty$
x^2-4	+	0	−	−	−	−	−	0	+
x^2+x	+	+	+	0	−	0	+	+	+
$\frac{x^2-4}{x^2+x}$	+	0	−	n.d.	+	n.d.	−	0	+

C.S. = $]-\infty, -2] \cup]-1, 0[\cup [2, +\infty[$

g) $\frac{1}{x-2} \geq 3$

$\frac{1}{x-2} \geq 3 \Leftrightarrow \frac{1}{x-2} - 3 \geq 0 \Leftrightarrow \frac{1-3x+6}{x-2} \geq 0 \Leftrightarrow \frac{7-3x}{x-2} \geq 0 \Leftrightarrow x \in \left]2, \frac{7}{3}\right]$

x	$-\infty$	2		$\frac{7}{3}$	$+\infty$
$7-3x$	$+$	$+$	$+$	0	$-$
$x-2$	$-$	0	$+$	$+$	$+$
$\frac{7-3x}{x-2}$	$-$	n.d.	$+$	0	$-$

h) $\frac{2x}{3-x} < 1$

$\frac{2x}{3-x} < 1 \Leftrightarrow \frac{2x}{3-x} - 1 < 0 \Leftrightarrow \frac{2x-3+x}{3-x} < 0 \Leftrightarrow \frac{3x-3}{3-x} < 0 \Leftrightarrow x \in]-\infty, 1[\cup]3, +\infty[$

x	$-\infty$	1		3	$+\infty$
$3x-3$	$-$	0	$+$	$+$	$+$
$3-x$	$+$	$+$	$+$	0	$-$
$\frac{3x-3}{3-x}$	$-$	0	$+$	n.d.	$-$

i) $\frac{x}{x-2} - \frac{2}{x^2-4} \geq \frac{1}{x+2}$

$\frac{x}{x-2} - \frac{2}{x^2-4} \geq \frac{1}{x+2} \Leftrightarrow \underset{(x+2)}{\frac{x}{x-2}} - \frac{2}{(x-2)(x+2)} - \underset{(x-2)}{\frac{1}{x+2}} \geq 0 \Leftrightarrow \frac{x^2+2x-2-x+2}{(x-2)(x+2)} \geq 0 \Leftrightarrow$

$\Leftrightarrow \frac{x^2+x}{x^2-4} \geq 0 \Leftrightarrow x \in]-\infty, -2[\cup [-1, 0] \cup]2, +\infty[$

x	$-\infty$	-2		-1		0		2	$+\infty$
x^2+x	$+$	$+$	$+$	0	$-$	0	$+$	$+$	$+$
x^2-4	$+$	0	$-$	$-$	$-$	$-$	$-$	0	$+$
$\frac{x^2+x}{x^2-4}$	$+$	n.d.	$-$	0	$+$	0	$-$	n.d.	$+$

j) $\frac{1}{3x-x^2} \leq \frac{2x}{(x-3)^2}$

$\frac{1}{3x-x^2} \leq \frac{2x}{(x-3)^2} \Leftrightarrow \frac{1}{-x(x-3)} - \frac{2x}{(x-3)^2} \leq 0 \Leftrightarrow \underset{(x-3)}{\frac{-1}{x(x-3)}} - \underset{(x)}{\frac{2x}{(x-3)^2}} \leq 0 \Leftrightarrow$

$\Leftrightarrow \frac{-x+3-2x^2}{x(x-3)^2} \leq 0 \underset{(*)}{\Leftrightarrow} \frac{-2\left(x+\frac{3}{2}\right)(x-1)}{x(x-3)^2} \leq 0 \Leftrightarrow \frac{(-2x-3)(x-1)}{x(x-3)^2} \leq 0 \Leftrightarrow$

$\Leftrightarrow x \in \left[-\frac{3}{2}, 0\right[\cup [1, 3[\cup]3, +\infty[$

x	$-\infty$	$-\frac{3}{2}$		0		1		3	$+\infty$
$-2x-3$	$+$	0	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$x-1$	$-$	$-$	$-$	$-$	$-$	0	$+$	$+$	$+$
x	$-$	$-$	$-$	0	$+$	$+$	$+$	$+$	$+$
$(x-3)^2$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	0	$+$
$\frac{(-2x-3)(x-1)}{x(x-3)^2}$	$+$	0	$-$	n.d.	$+$	0	$-$	n.d.	$-$

(*) $-2x^2 - x + 3 = 0 \Leftrightarrow x = \frac{1 \pm \sqrt{1-4\times(-2)\times 3}}{2\times(-2)} \Leftrightarrow x = \frac{1\pm 5}{-4} \Leftrightarrow$

$\Leftrightarrow x = 1 \vee x = -\frac{3}{2}$

4. Considera a família de funções *f* definidas por $f(x) = \frac{x^2-9}{x+k}$ com $k \in \mathbb{R}$.

Determina os valores de k para os quais a função *f* :

a) tem apenas um zero;

5.1. $f(x)=0 \Leftrightarrow \frac{x^2-9}{x+k}=0 \Leftrightarrow x^2-9=0 \quad \wedge \quad x+k \neq 0 \Leftrightarrow (x=3 \quad \vee \quad x=-3) \quad \wedge \quad x \neq -k$.

A função *f* tem pelo menos um zero se $-k=3 \quad \vee \quad -k=-3$, ou seja, se $k \in \{-3,3\}$.

b) tem dois zeros;

A função *f* tem dois zeros se $k \in \mathbb{R} \setminus \{-3,3\}$.

c) não tem zeros.

Não existe nenhum valor de *k* para o qual a função *f* não tenha zeros porque a função *f* tem sempre pelo menos um zero, qualquer que seja o valor de *k*.